



THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

JANUARY – APRIL 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 101: FUNDAMENTALS OF ACTUARIAL MATHEMATICS I

Date: APRIL 2019

Duration: 3 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

- Q1. a) An investor makes an initial investment of £5,000 and is credited with £500 interest at the end of the year. What is the effective rate of interest and the value of i ? **[2 marks]**
- b) A rate of interest of 4%p.a convertible monthly is equivalent to what annual effective rate of discount? **[4 marks]**
- c) Working in time units of years, we will let the nominal rate of interest per year on monthly transactions over the next year be 12%. If £100 is invested at time 0, how much is it worth at the end of the first month and at the end of the third month? **[4 marks]**
- d) Prove algebraically and by general reasoning that $\ddot{a}_{\overline{n}|} = a_{\overline{n-1}|} + 1$ **[4 marks]**
- e) Derive an expression for annuity in arrear **[4 marks]**
- f) Derive an expression for an increasing annuity in arrear **[4 marks]**
- g) You deposit 1, 000 today and 2, 000 in five years into a fund that pays simple discount at 5% per annum. Your friend makes the same deposits into another fund but at times n and $2n$ respectively and the fund credits interest at an annual effective rate of interest of 10%. At the end of 10 years, the accumulated value of your friend's deposits is exactly the same as the accumulated value of your deposits. Calculate n **[6 marks]**

- h) Under what circumstances are the simple rate of interest and compound rate of interest applied? **[2 marks]**
- Q2. a) Given $i=8\%$, calculate $i^4 \wedge d^{12}$ **[6 marks]**
- b) If $a_{\overline{n}|} = 5$ and $s_{\overline{n}|} = 15$, find i . **[5marks]**
- c) A man makes payments into an investment account of \$200 at time 5, \$190 at time 6, \$180 at time 7, and so on until a payment of \$100 at time 15. Assuming an annual effective rate of interest of 3.5%, calculate:
- The present value of the payments at time 4
 - The present value of the payments at time 0
 - The accumulated value of the payments at time 15 **[9 marks]**
- Q3. a) A loan is to be repaid by annual instalments of P at the end of each year for 10 years. The total principal repaid in the first 3 years is *KSh. 29,035* and the total principal repaid in the last 3 years is *KSh. 40,855*. Determine the total amount of interest paid during the life of the loan. **[8 marks]**
- b) An investment of £1,000 made at time 0 is accumulated at the following rates: 8% per annum simple for two years, followed by a rate of discount of 6% per annum convertible monthly for two years. Calculate the accumulated amount of the investment after 4 years. **[5 marks]**
- c) 4600 is invested at time 0 and the proceeds at time 10 are 8200. Calculate $A(7,10)$ if you are given $A(0,9)=1.8$, $A(2,4)=1.1$, $A(2,7)=1.32$ and $A(4,9)=1.45$ **[7 marks]**
- Q4. a) Find the present value as at 1 January 2004 of a series of payments of £100 payable on the first day of each month during 2005, 2006 and 2007, assuming an effective rate of interest of 8% per annum. **[3 marks]**
- b) Show that $i^{(p)} = p[(1+i)^{\frac{1}{p}} - 1]$ and calculate $i^{(5)}$ and $d^{(4)}$ if $i^{(3)} = 12\%$. **[10 marks]**
- c) A company expects to receive £10,000 from a customer. The amount will be split into ten equal installments. The first installment is in one year's time and the remaining payments will be made at three year intervals. Calculate the present value of the future receipts at 7% *pa*. **[3 marks]**
- d) Define a simple increasing annuity. **[1 mark]**
- e) Find the present value as at 1 January 2005 of a series of 10 annual payments starting at £500 on 1 January 2006 and increasing by £100 each year. Assume an effective rate of interest of 8% per annum **[3 marks]**

- Q5. a) Jane deposits 1,000 in her savings account in year 1, 1,500 in year 2, 2,000 in year 3, and 4,000 in year 4. Her bank pays interest rate of 8% per annum effective. How much is in her account at the end of 10 years inclusive. **[5 marks]**
- b) At what simple rate of interest will *KSh.* 5,000 accumulate to *KSh.* 8,200 in 4.5 years? **[4 marks]**
- c) A loan of £16,000 is repayable by ten equal annual payments. The annual effective rate of interest is 4%. Calculate:
- i) The interest element of the 4th payment
 - ii) The capital element of the 7th payment
 - iii) The capital repaid in the last five years of the loan
 - iv) The total interest paid over the whole loan. **[11 marks]**

END