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MAIN EXAMINATION

JANUARY – APRIL 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 203:OPERATIONS RESEARCH II

Date: APRIL 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define the following terms as used in Operations Research

i) Feasible solution **(1mark)**

ii) Non-degenerate basic feasible solution **(2marks)**

iii) Queuing system **(2marks)**

iv) Optimal solution **(2marks)**

b) Give the mathematical formulation of an Assignment Problem **(4marks)**

c) State two differences between Transport Problem and Assignment Problem **(2marks)**

d) Distinguish the terms Earliest start time(EST) and Latest start time (LST)

(3marks)

e) Construct a network for the project whose activities and their precedence relationships are given below;

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	-	-	A,B	B	B	A,B	F,D	F, D	C.G

(5marks)

f) Customers arrive at a bank at a Poisson rate λ / hr. Suppose that two customers arrive during the first hour. What is the probability that both arrived during the first 20 minutes? **(5marks)**

g) Consider the queuing model $(M / M / 1):(\infty / FCFS)$
Explain clearly the meaning of each symbol in the model **(4marks)**

Q2. a) The following table shows the jobs of a network along with their time estimates. The time estimates are in days:

Job	1-2	1-6	2-3	2-4	3-5	4-5	5-8	6-7	7-8
a	3	2	6	2	5	3	1	3	4
m	6	5	12	5	11	6	4	9	19
b	15	14	30	8	17	15	7	27	28

i) Draw the project network **(5marks)**
ii) Find the critical path **(3marks)**
iii) Find the probability that the project is completed in 31 days.

(4marks)

b) A company has 5 jobs to be done on five machines. Any job can done on any machine. The cost $(Ksh'000)$ of doing the jobs in different machines are given below. Assign the jobs for different machines so as to minimize the total cost.

Jobs	**Machines**				
	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

(8marks)

Q3. a) Find the initial basic feasible solution for the following transportation problem using the Vogel's Approximation Method

Destination						
Factory		D_1	D_2	D_3	D_4	**Supply**
	F_1	3	3	4	1	100
	F_2	4	2	4	2	125
	F_3	1	5	3	2	75
	Demand	120	80	75	25	300

(10arks)

b) Arrivals in a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a telephone call is assumed to be distributed exponentially with mean 3 minutes.

i) What is the probability that a person arriving at the booth will have to wait? **(3marks)**

ii) What is the average length of the queue that forms from time to time? **(3marks)**

iii) The telephone department will install a second booth when convinced that an arrival would except to have to wait at least three minutes for the phone. By how much must the flow of arrivals be increased in order to justify second booth? **(4marks)**

Q4. a) A company is producing a single product and is selling it through fire agencies situated in different cities. All of a sudden there is demand for the product in another five cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distances is minimized. The distance (in km's) between the surplus and the deficit cities are given in the following distance matrix.

Surplus cities/Deficit cities	**Programmes**				
Programmers	**I**	**II**	**III**	**IV**	**V**
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	55	70	80	105

Determine the optimum assignment schedule. **(8 marks)**

b) Define the following terms as used in

Queuing Theory; Reneging, Jockeying, Balking

(6marks)

c) In $(M/M/1):(\infty/FCFS)$ Queuing model, $p_n = (1-\rho)\rho^n$ where $\rho = \lambda/\mu < 1, n = 1, 2, ...$. Show that $L_s = \frac{\rho}{1-\rho}$ where L_s is the expected number of units in the system. **(6marks)**

Q5. a) Tasks A,B,...,H ,I constitute a project. The notation X<Y means that the task X must be completed before Yis started.

With the notation,

$A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I$

Draw a graph to represent the sequence of tasks and find the minimum time of completion of the project, when the time (in days) of completion of each task is as follows.

The above constraints can be given in the following table

Task	A	B	C	D	E	F	G	H	I
Time (days)	8	10	8	10	16	17	18	14	9

(12marks)

b) Use dynamic programming to solve the LPP

$Max.Z = x_1 + 9x_2$

Subject to the constraints

$2x_1 + x_2 \leq 25$

$x_2 \leq 11$

$x_1, x_2 \geq 0$

(8marks)

END