# A. M. E. C. E. A <br> MAIN EXAMINATION <br> JANUARY - APRIL 2019 TRIMESTER <br> FACULTY OF COMMERCE <br> DEPARTMENT OF ACCOUNTING AND FINANCE <br> ODEL / EVENING PROGRAMME <br> CMS 311: BUSINESS STATISTICS 

Date: APRIL 2019
Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and ANY OTHER TWO Questions

Q1. a) Why linear regression sometime is referred to as least squares?
b) How does the correlation coefficient relate to the slope of the regression line?
c) A student wants to compute the correlation coefficient with two data pairs. What value or values of $r$ should he expect? Why?
d) Interesting data given as

| $X$ | $Y$ |
| :--- | :--- |
| 72 | 45 |
| 73 | 38 |
| 75 | 41 |
| 76 | 35 |
| 77 | 31 |
| 78 | 40 |
| 78 |  |
| 79 | 25 |
| 80 | 32 |
| 80 | 36 |
| 81 | 29 |


| 84 | 26 |
| :--- | :--- |
| 85 | 32 |
| 86 | 28 |
| 88 | 27 |

c) Compute the coefficients of the linear regression line, $y=b_{1}+b_{0}$

Q2. I) In each of the 4 races, the democrats have $60 \%$ chance of winning. Assuming that the races are independent of each other, what is the probability that:
a) The Democrats will win 0 races, 1 race, 2 races, 3 races, or 4 races?
(4 marks)
b) The Democrats will win at least 1 race.
(3 marks)
c) The Democrats will win a majority of the races.
II) A discrete random variable (RV) has the following Probability Distribution:

|  | 5 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $X$ | 1 | 2 | 4 | 5 | 8 |
| $\operatorname{Pr}(x)$ | 0.20 | 0.25 |  | 0.30 | 0.10 |

Required:
i) Find the $\operatorname{Pr}(4)$
ii) Find the $((\operatorname{Pr}(x)=2)$ or $(\operatorname{Pr}(x)=4))$.
iii) Find the $\operatorname{Pr}(x \leq 4)$
iv) Find the $\operatorname{Pr}(x<4)$

Q3. a) The table below shows the political affiliation of American voters and the proportion favouring or opposing the death penalty within the 6 categories defined by three values of party affiliation and 2 opinions.

| Death Penalty Opinion |  |  |
| :--- | :--- | :--- |
| Party | Favour | Oppose |
| Republican | 0.26 | 0.04 |
| Demorrat | 0.12 | 0.24 |
| Other | 0.24 | 0.01 |

a) What is the probability that a randomly chosen voter favours the death penalty?
b) What is the Probability that a different randomly chosen voter is a Republican?
b) The regional chairman of the Muscular Dystrophy Association is typing to estimate the amount each caller will pledge during the annual MDA telethon. Using data gathered over the past 10 years, she has computed the following
probabilities of various pledge amounts. Draw a graph illustrating this probability distribution.

| Dollars pledged | 25 | 50 | 75 | 100 | 125 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.45 | 0.25 | 0.15 | 0.10 | 0.05 |

Q4. a) The mean of a Binomial distribution is 40 and the standard deviation 6. Calculate $\mathrm{n}, \mathrm{p}$ and q .
b) The screws produced by a certain machine were checked by examining number of defectives in a sample of 12. The following table shows the distribution of 128 samples according to the number of defective items they contained.

| No. of defectives in a <br> sample of 12 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No of samples | 7 | 6 | 19 | 35 | 30 | 23 | 7 | 1 | 128 |

Required:
i) Fit a binomial distribution and find the expected frequencies if the chance of machine being defective is 0.5 .
ii) Find the mean and standard deviation of the fitted distribution.

## CMS 311 BUSINESS STATISTICS FORMULAE

## CORRELATION AND REGRESSION

Covariance $\left(\operatorname{cov}_{(x y)}\right.$ or $\left.S_{x y}\right)=\frac{1}{N} \sum\left(x_{i}-\ddot{X}\right)\left(y_{i}-\dot{Y}\right)$
$\operatorname{cov}_{(x y)}$ or $S_{x y}=\left(\frac{1}{N} \sum x_{i} y_{i}\right)-\ddot{X} \dot{Y}$
Coefficient of Correlation $\left(r_{x y}\right)=\frac{S x y}{S x S y}$

$$
r=\frac{n \sum x y-\left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^{2}\right)-\left(\sum x\right)^{2}} \sqrt{n\left(\sum y^{2}\right)-\left(\sum y\right)^{2}}}
$$

Rank correlation coefficient or spearman's rank correlation coefficient $\left(r_{s}\right)$

$$
\rho=1-\frac{6 \sum d_{i}^{2}}{n\left(n^{2}-1\right)}
$$

Kendall Rank correlation

$$
\tau=\frac{n_{c}-n_{d}}{\frac{1}{2} n(n-1)}
$$

Pearson r correlation
$\gamma=\frac{\mathrm{N} \sum \mathrm{xy}-\sum(\mathrm{x})(\mathrm{y})}{\sqrt{\left.N \sum x^{2}-\sum\left(x^{2}\right)\right]\left[N \sum y^{2}-\sum\left(y^{2}\right)\right]}}$
Method of least squares

$$
\begin{aligned}
& \sum \mathrm{y}=\mathrm{na}+\mathrm{b} \sum \mathrm{x}_{\mathrm{i}} \\
& \sum \mathrm{y}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}=\mathrm{a} \sum \mathrm{x}_{\mathrm{i}}+\mathrm{b} \sum \mathrm{x}_{\mathrm{i}}{ }^{2} \\
& \sum \mathrm{x}_{\mathrm{i}}=\mathrm{na}+\mathrm{b} \sum \mathrm{y}_{\mathrm{i}} \\
& \sum \mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}=\mathrm{a} \sum \mathrm{y}_{\mathrm{i}}+\mathrm{b} \sum \mathrm{y}_{\mathrm{i}}{ }^{2}
\end{aligned} \quad \begin{aligned}
& \mathrm{b}=\frac{n\left(\sum X Y\right)-\left(\sum X\right)\left(\sum Y\right)}{n\left(\sum X^{2}\right)-\left(\sum X\right)^{2}} \\
& \mathrm{a}=\frac{\left(\sum Y\right)-b\left(\sum X\right)}{n} \\
& L S M A=a+b X \\
& \mathrm{~b}=\mathrm{r}_{\mathrm{xy}}-\frac{S y}{S X} \\
& \mathrm{a}=\dot{\mathrm{Y}}-\mathrm{b} \ddot{\mathrm{X}}
\end{aligned}
$$

## PARAMETERS

- Population mean $=\mu=\left(\Sigma X_{i}\right) / N$
- Population standard deviation $=\sigma=\operatorname{sqrt}\left[\Sigma\left(X_{i}-\mu\right)^{2} / N\right]$
- Population variance $=\sigma^{2}=\Sigma\left(X_{i}-\mu\right)^{2} / N$
- Variance of population proportion $=\sigma_{P}{ }^{2}=P Q / n$
- Standardized score $=Z=(X-\mu) / \sigma$


## Statistics

Unless otherwise noted, these formulas assume simple random sampling.

- Sample mean $=x=\left(\sum x_{i}\right) / n$
- Sample standard deviation $=s=\operatorname{sqrt}\left[\Sigma\left(x_{i}-x\right)^{2} /(n-1)\right]$
- Sample variance $=s^{2}=\Sigma\left(x_{i}-x\right)^{2} /(n-1)$
- Variance of sample proportion $=s_{p}{ }^{2}=p q /(n-1)$


## Counting

- n factorial: $\mathrm{n}!=\mathrm{n}$ * $(\mathrm{n}-1)$ * $(\mathrm{n}-2)$ * ... * 3 * 2 * 1 . By convention, $0!=1$.
- Permutations of $n$ things, taken $r$ at a time: ${ }_{n} P_{r}=n!/(n-r)$ !
- Combinations of $n$ things, taken $r$ at a time: ${ }_{n} C_{r}=n!/ r!(n-r)!={ }_{n} P_{r} / r!$


## Probability

- Rule of addition: $P(A \cup B)=P(A)+P(B)-P(A \cap B)$
- Rule of multiplication: $P(A \cap B)=P(A) P(B \mid A)$
- Rule of subtraction: $P\left(A^{\prime}\right)=1-P(A)$


## Random Variables

In the following formulas, $X$ and $Y$ are random variables, and $a$ and $b$ are constants.

- Expected value of $X=E(X)=\mu_{x}=\Sigma\left[x_{i}{ }^{*} P\left(x_{i}\right)\right]$
- Variance of $X=\operatorname{Var}(X)=\sigma^{2}=\Sigma\left[x_{i}-E(x)\right]^{2} * P\left(x_{i}\right)=\Sigma\left[x_{i}-\mu_{x}\right]^{2} P\left(x_{i}\right)$
- Normal random variable $=z$-score $=z=(X-\mu) / \sigma$
- Chi-square statistic $=X^{2}=\left[(n-1){ }^{*} s^{2}\right] / \sigma^{2}$
- f statistic $=f=\left[s_{1}{ }^{2} / \sigma_{1}{ }^{2}\right] /\left[s_{2}{ }^{2} / \sigma_{2}{ }^{2}\right]$
- Expected value of sum of random variables $=E(X+Y)=E(X)+E(Y)$
- Expected value of difference between random variables $=E(X-Y)=E(X)-E(Y)$


## Sampling Distributions

- Mean of sampling distribution of the mean $=\mu_{x}=\mu$
- Mean of sampling distribution of the proportion $=\mu_{p}=P$
- Standard deviation of proportion $=\sigma_{p}=\operatorname{sqrt}\left[P^{*}(1-P) / n\right]=\operatorname{sqrt}(P Q / n)$
- Standard deviation of the mean $=\sigma_{x}=\sigma / \operatorname{sqrt}(n)$
- Standard deviation of difference of sample means $=\sigma_{d}=\operatorname{sqrt}\left[\left(\sigma_{1}{ }^{2} / n_{1}\right)+\left(\sigma_{2}{ }^{2} / n_{2}\right)\right.$ ]


## Standard Error

- Standard error of proportion $=\mathrm{SE}_{\mathrm{p}}=\mathrm{s}_{\mathrm{p}}=\operatorname{sqrt}\left[\mathrm{p}^{*}(1-\mathrm{p}) / \mathrm{n}\right]=\operatorname{sqrt}(\mathrm{pq} / \mathrm{n})$
- Standard error of difference for proportions $=\operatorname{SE}_{p}=s_{p}=\operatorname{sqrt}\left\{p^{*}(1-p)\right.$ * $\left[\left(1 / n_{1}\right)\right.$ $\left.\left.+\left(1 / n_{2}\right)\right]\right\}$
- Standard error of the mean $=\mathrm{SE}_{\mathrm{x}}=\mathrm{s}_{\mathrm{x}}=\mathrm{s} / \operatorname{sqrt}(\mathrm{n})$
- Standard error of difference of sample means $=\operatorname{SE}_{\mathrm{d}}=\mathrm{s}_{\mathrm{d}}=\operatorname{sqrt}\left[\left(\mathrm{s}_{1}{ }^{2} / \mathrm{n}_{1}\right)+\left(\mathrm{s}_{2}{ }^{2} /\right.\right.$ $\mathrm{n}_{2}$ )]
- Standard error of difference of paired sample means $=$ SE $_{d}=s_{d}=\{$ sqrt $\left[\left(\Sigma\left(d_{i}-d\right)^{2} /(n-1)\right]\right\} / \operatorname{sqrt}(n)$


## Discrete Probability Distributions

- Binomial formula: $\mathrm{P}(\mathrm{X}=\mathrm{x})=\mathrm{b}(x ; n, P)={ }_{\mathrm{n}} \mathrm{C}_{\mathrm{x}}{ }^{*} \mathrm{P}^{\mathrm{x}}$ * $(1-\mathrm{P})^{\mathrm{n}-\mathrm{x}}={ }_{\mathrm{n}} \mathrm{C}_{\mathrm{x}}{ }^{*} \mathrm{P}^{\mathrm{x}}{ }^{*} \mathrm{Q}^{\mathrm{n}-\mathrm{x}}$
- Mean of binomial distribution $=\mu_{\mathrm{x}}=\mathrm{n}$ * P
- Variance of binomial distribution $=\sigma_{x}^{2}=n^{*} P^{*}(1-P)$
- Negative Binomial formula: $\mathrm{P}(\mathrm{X}=\mathrm{x})=\mathrm{b} *(x ; r, P)={ }_{x-1} \mathrm{C}_{\mathrm{r}-1}$ * $\mathrm{P}^{r} *(1-\mathrm{P})^{\mathrm{x}-\mathrm{r}}$
- Mean of negative binomial distribution $=\mu_{x}=r Q / P$
- Variance of negative binomial distribution $=\sigma_{x}^{2}=r$ * $Q / P^{2}$
- Poisson formula: $\mathrm{P}(x ; \mu)=\left(\mathrm{e}^{-\mu}\right)\left(\mu^{x}\right) / x$ !
- Mean of Poisson distribution $=\mu_{x}=\mu$
- Variance of Poisson distribution $=\sigma_{x}^{2}=\mu$

Multinomial formula: $P=\left[n!/\left(n_{1}!{ }^{*} n_{2}!{ }^{*} \ldots n_{k}!\right)\right]^{*}\left(p_{1}{ }^{n}{ }_{1}{ }^{*} p_{2}{ }^{n}{ }_{2}{ }^{*} \ldots{ }^{*} p_{k}{ }^{n} k\right)$
*END*

