THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

ACS 201: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

Date: JULY 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1

A. i. In the context of random variables, define T_X and K_x **2mks**

ii. A person is aged exactly 45 years old. Suppose that she dies when she is aged 84 years and 150 days old. What are the values of T_{45} AND K_{45} for this person? **2mks**

B. Define the UDD assumption and hence prove that ${}_tq_x \approx tq_x$ **6 mks**

C. Discuss the three forms of premium frequencies that insurance companies use in their contract pricing **6mks**

D. Calculate $A_{50:\overline{3}|}$

Basis:

Mortality $q_{50} = 0.05$

$q_{51} = 0.06$

$q_{51+t} = 1.1q_{50+t}$ for $t \geq 1$

Interest 6% p.a. **7mks**

E. Calculate $\ddot{a}_{40:4\rceil}$

Basis:

From the following life table extract

X	l_x
40	100,000
41	99,300
42	98,200
43	96,600
44	94,600

Interest 4.5% per annum **7mks**

Q2

A. Pove that

$$A_{[x]} = vq_{[x]} + vp_{[x]}A_{[x]+1}$$

4mks

Hence orotherwise, using AM92 tables at 6% p.a interest, compute

$A_{[50]+1:9\rceil}$ **7mks**

B. The table below is part of a mortality table used by a life insurance company to calculate survival probabilities for a special type of life insurance policy.

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}
51	1537	1517	1502	1492	1483
52	1532	1512	1497	1487	1477
53	1525	1505	1490	1480	1470
54	1517	1499	1484	1474	1462
55	1512	1492	1477	1467	1453

i. Calculate the probability that a policy holder who was accepted for insurance exactly 2 years ago and is now aged exactly 55 will die at age 57 next birthday. **4 mks**

ii. Calculate the corresponding probability for an individual of the same age who has been a policyholder for many years. **3 mks**

iii. Comment on your answers to (i) and (ii). **2 mks**

Q3

A. Discuss five factors to consider when performing an insurance valuation. **10mks**

B. If T_x and K_x are random variables measuring the complete and curtate future lifetimes, respectively, for a life aged x, write down expressions for the following symbols in terms of expected values.

I. A_x

II. $\overline{A}^{1}_{x:\overline{n}|}$

III. $A^{\ 1}_{x:\overline{n}|}$

IV. $\overline{a}_x$

V. $\ddot{a}_{x:\overline{n}|}$

5mks

C. A whole life assurance provides a benefit of 100,000 payable immediately on the death of a male life who is now aged 45 exact.

Calculate, showing all your workings: the EPV of this policy.

Basis:

Mortality AM92 Select

Rate of interest 4% p.a. **5mks**

Q4

A. Explain what the following represent:

i. $l_{[x-1]+1}$

ii. d_x **2 mks**

B. Calculate the values for the following functions, assuming AM92 mortality at 4% pa interest:

i. $\ddot{a}_{23|18|}$

ii. $\frac{D_{50}}{D_{40}} a_{50}$ **8 mks**

C. You are given that $p_{80} = 0.888$. Estimate ${}_{0.5}p_{80}$ assuming:

i. A uniform distribution of death between integer ages

ii. A constant force of mortality between integer ages **4mks**

D. Calculate the exact value of $\overline{A}^{1}_{70:\overline{1}|}$ assuming the force of mortality is constant between consecutive integer ages.

Basis: Mortality: ELT15 (Males)

Interest: 4.5% per annum **6 mks**

Q5

A. Calculate: ${}_{12}p_{[60]+1}$

Basis: AM92 Mortality **2mks**

B. A graph of $f_0(t)$, the probability density function for the random future lifetime , is plotted on the vertical axis, with t plotted on the horizontal axis, for data taken from the English Life Table No. 15(Males)

You are given that $f_0(t) = {}_tp_0\mu_t$. You observe that the graph rises to a peak at around t=80 and then falls. Explain why the graph falls at around t=80 **3mks**

C. The mortality of a certain population is governed by the life table function $l_x = 100\text{-}x$, $0 \leq x < 100$. Calculate the values of the following expressions:

i. μ_{30}

ii. ${}_{10}p_{30}$

iii. $P(T_{30} > 20)$

iv. $P(K_{30} = 20)$

v. $\overset{o}{e}_{30}$ **9mks**

D. An assurance contract provides a death benefit of £1,250 payable immediately on death.
The following basis is used:
Force of mortality: $\mu_x = 0.045$ for all x
Force of interest: $\delta = 0.045$
Calculate the EPV. **6mks**

END