THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

ACS 301: ACTUARIAL MATHEMATICS II

Date: JULY 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Question One (30 marks)

a.) Define the meaning of competing risks

(2 marks)

b.) Explain the difference between a profit vector and a profit signature.

(4 marks)

c.) (i) Define the meaning of zeroisation in the context of unit linked policy.

(2 marks)

(ii) Explain why an insurance company might choose to zeroise the above profit vector.

(2 marks)

d.) A five year unit-linked policy issued by an insurance company to a life aged 60 exact has the following profit vector:

(751.25, -321.06, -267.57, -192.05, 201.75)

Calculate, showing all your workings, the net present value of the profits of this policy after zeroisation.

Basis:

Mortality AM92 Ultimate

Rate of interest on non-unit fund cash flows 3.5% per annum

Risk discount rate 6.0% per annum (8 marks)

e.) Why study actuarial mathematics in higher education?(4 marks)

e.) A certain population is subject to three modes of decrement: α, β and γ. Write down an expression for ¿in terms of the single decrement table probabilities q_x^{α}, q_x^{β}, and q_x^{γ}, assuming each of the three modes of decrement is uniformly distributed over the year of age x to $x + 1$ in the corresponding single decrement table (2 marks)

f.) A life insurance company writes policies that provide income during periods of disability. Draw the transition state model for these policies labelling your diagram carefully. (5 marks)

Question Two (20 marks)

A life insurance company issues a 3-year unit-linked endowment assurance contract to a female life aged 60 exact under which level premiums of Kshs 5,000 per annum are payable in advance. In the first year, 85% of the premium is allocated to units and 104% in the second and third years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.75% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of Kshs 20,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On survival to the end of the term, the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units but does not set up non-unit reserves.

It uses the following assumptions in carrying out profit tests of this contract:

Mortality: AM92 Ultimate

Surrenders: None

Expenses: Initial: Kshs 600

Renewal: Kshs 100 at the start of each of the second and third policy years

Unit fund growth rate: 6% per annum

Non-unit fund interest rate: 4% per annum

Risk discount rate: 10% per annum

(i.) Calculate the expected net present value of the profit on this contract. (18 marks)

(ii.) State, with a reason, what the effect would be on the profit if the insurance company did hold non-unit reserves to zeroise negative cashflows, assuming it used a discount rate of 4% per annum for calculating those reserves. (You do not need to perform any further calculations.) (2 marks)

Question Three (20 marks)

a.) A company provides its employees with a benefit on disability before age 65. The benefit is a life annuity of 50% of salary at the date of disability.

(i) Draw and label a transition state diagram for this benefit.

(8 marks)

(ii) Derive a formula for the expected present value of this benefit for a life aged x with a current annual salary of 20,000.

(7 marks)

b.) A reversionary annuity is payable continuously beginning on the death of a life aged x to an annuitant aged y. Derive an expression for the present value of the reversionary annuity using random variables for the future lifetimes.

(5 marks)

Question Four (20 marks)

A life insurance company issues a large number of 4-year unit-linked endowment assurance policies to lives aged 65 exact. Level premiums are payable annually in advance until maturity or earlier death.

The company has performed a profit test on these policies and the profit vector per policy sold, ignoring surrenders, is as follows:

(185.21, –121.52, –5.28, 12.95)

(i) Calculate the profit signature per policy sold if negative non-unit fund cash flows are zeroised.

(6 marks)

The company now wishes to allow for surrenders in its calculations. It assumes that at the end of the first and second policy years only, 3% of the surviving policyholders will surrender. Surrender values are equal to the bid value of units held (after deduction of the fund management charge) less a surrender penalty of 50.

(ii) Calculate the revised profit signature per policy sold after allowing for surrenders if negative non-unit cash flows are zeroised.

(12 marks)

(iii) Calculate the net present value of the revised profit signature in part (ii), using a risk discount rate of 8% per annum.

(2 marks)

Basis:

Mortality AM92 Ultimate

Interest earned on non-unit cash flows 5% per annum fund

Expenses Ignore

Question Five (20 marks)

a.) The random variable *Txy*represents the time to failure of the joint-life status ($x\ y$). (x) is subject to a constant force of mortality of 0.02 and (y) is subject to a constant force of mortality of 0.03. (x) and (y) are independent with respect to mortality.
Calculate the value of $E[Txy]$.

(8 marks)

b.) The table below shows an extract from a study of mortality for a small country:

Age	*Population*	*Number of deaths*
60	9,950	52
61	8,020	68
62	6,997	73

(i) Show that the standardised mortality ratio using ELT15 (Males) as the standard population is 0.5. It can be assumed that the age definition of the country matches that of ELT15 (Males).

(8 marks)

(ii) Explain what the result in part (i) indicates.

(4 marks)

END