THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157

00200 Nairobi - KENYA

Telephone: 891601-6

MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

MAT 161: PROBABILITY AND STATISTICS II

Date: JULY 2019	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

QUESTION ONE

a) Define a random variable. (1mark)

b) A discrete random variable has a pmf give by the table below.

x	0	1	2	3	4
$P(X=x)$	k	2k	5k	10k	17k

Find the value of the constant k. hence compute $p(1 \leq x < 3)$ (5marks)

c) Let x be a random variable with probability distribution $P(X=x)$. Prove that:

i. $E(c)=c$ Where c is an arbitrary constant. (2marks)

ii. $E(ax+b)=a\mu+b$ Where a and b is an arbitrary constant. (3marks)

d) John travels always by plane. From past experience, he feels that take off time is uniformly distributed between 80 and 120 minutes after check in. determine the probability that he waits more than 105 minutes for takeoff after check in. (4marks)

e) A continuous random variable x has pdf given by

$$f(x)=\begin{cases} 0.5x, 0 \leq x \leq 2 \\ 0, elsewhere \end{cases}$$

Find the mean and standard deviation of x (9marks)

f) In an examination, the average mark was 76.5 and the standard deviation was 9.5. If 5% of the class score grade A and the marks are assumed to follow a normal distribution, what is the lowest possible grade A mark. (6marks)

QUESTION TWO

a) Let x be a random variable with mean $E(X)=\mu$ prove that

i. $var(X) = E(X-\mu)^2 = E(X^2) - \mu^2$ (4marks)

ii. $var(ax+b) = a^2 var(X)$ (4marks)

b) Let Y be a random variable with pdf $f(y) = \begin{cases} \frac{3}{64} y^2 (4-y), 0 \le y \le 4 \\ 0\, elsewhere \end{cases}$

i. Find the expected value and variance of *Y*. (4marks)

ii. Let $X = 300Y + 50$ find $E(X)$ and $var(X)$ (2marks)

iii. Find $p(X > 750)$ (4marks)

c) Write in terms of the moment generating function and its derivatives an expression for the mean and variance of a random variable. (2marks)

QUESTION THREE

a) A biased coin is tossed 6 times. The probability of head on any toss is 0.3. let x denote the number of heads that come up. Calculate:

i. $p(X=3)$ (2marks)

ii. $p(1 < X \le 5)$ (4marks)

iii. What is the mean and standard deviation of the heads that turned out? (3marks)

b) Customers arrive at a checkout counter according to a poisson distribution at an average of 7 per hour. During a given hour, what are the probabilities that:

i. No more than 3 customers arrive (5marks)

ii. At least 2 customers arrive (4marks)

iii. Exactly 5 customers arrive (2marks)

QUESTION FOUR

a) Define moment generating function. (1mark)

b) Given that the mgf of a non negative discrete random variable x is $M_x(t) = \left(\frac{2}{3}e^t + \frac{1}{3}\right)^9$

i. State the distribution of x (1mark)

ii. Show that $p(X \ge 2) = 1 - 19(3^{-9})$ (3marks)

iii. Find $E(X)$ and $var(X)$ (2marks)

c) A random variable x has a probability generating function pgf given by $G_x(t) = e^{4(t-1)}$ determine its probability distribution. Hence or otherwise compute $p(X \ge 1)$ (6marks)

d) Let x be a discrete random variable with pmf given by

$$f(x) = \begin{cases} \frac{1}{20}(1+x), 0 \le x \le 2 \\ 0, elsewhere \end{cases}$$

Determine the cdf of x (7marks)

QUESTION FIVE

a) Consider the joint distribution of x and y given in the form of the table below. The cell (i, j) corresponds to the joint probability that $x=i$ and $y=j$. $(i=1,2,3\ j=1,2,3)$

		x		
y		1	2	3
	1	0	$\frac{1}{6}$	$\frac{1}{6}$
	2	$\frac{1}{6}$	0	$\frac{1}{6}$
	3	$\frac{1}{6}$	$\frac{1}{6}$	0

i. Show that it is a probability distribution. (1mark)

ii. Find the marginal probability of x and y. ie $f_x ¿ f_y$ (2marks)

iii. Compute the correlation coefficient between x and y (7marks)

b) If x is a random variable with pdf given by $f(x)=\begin{cases} \frac{1}{2\sqrt{3}}, -\sqrt{3} \le x \le \sqrt{3} \\ 0\, elsewhere \end{cases}$

Using Chebyshevs inequality, find the upper bound of $p\left[|x-\mu_x| \ge 1.5\delta_x\right]$ where and is the mean and standard deviation of x respectively. What is the exact probability? (10marks)

END