THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

MAT 204: LINEAR ALGEBRA II

Date: JULY 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

1. a). Describe a linear mapping. (3 marks)

   b). Let $L:R_2 \rightarrow R_2$ be defined by $L[(u_1, u_2)]=[u_1{}^2 2u_2.]$ Is L a linear transformation? (6 marks)

   c). State five properties of determinants. (5 marks)

   d). Let $L:R_2 \rightarrow R_2$ be the linear operator defined by $L\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}=\begin{pmatrix} -a_2 \\ a_1 \end{pmatrix}$. Find the Eigen values. (5 marks)

   e). State Cayley-Hamilton theorem. (2 marks)

   f). Explain the term invariant subspace. (2 marks)

   g). Compute the area of triangle T with vertices $(-1,4),(3,1) \wedge (2,6)$. (5 marks)

   h). suppose that **u** is a vector in R^n. Show that $u \pm u=O$. (2marks)

2. a). Let $L:P_1 \rightarrow P_2$ be defined by
   $L[P(t)]=tp(t)$, show that L is a linear transformation. (5 marks)

   b). Show that $s=\{t^2+t, t+1, t-1\}$ is a basis for p_2 under the mapping
   $L:P_2 \rightarrow R^3$ defined by $L\{at^2+bt+c\}=\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. (7 marks)

   c). Let $L:R_4 \rightarrow R_2$ be a linear transformation and let $s=\{v_1, v_2, v_3, v_4\}$ be a basis for R_4, where

$v_1=[1 \quad 0 \quad 1 \quad 0], v_2=[0 \quad 1 \quad -1 \quad 2], v_3=[0 \quad 2 \quad 2 \quad 1] \wedge v_4=[1 \quad 0 \quad 0 \quad 1]$. Suppose that $L(v_1)=[1 \quad 2], L(v_2)=[0 \quad 3], L(v_3)=[0 \quad 0]$ and $L(v_4)=[2 \quad 0]$. If $v=[3 \quad -5 \quad -5 \quad 0]$. Find $L[v]$. (7 marks)

3. a). Let $L:P_2 \to P_1$ be defined by $L[P(t)]=p'(t)$ and consider the ordered bases $s=\{t^2, t, 1\}$ and $T[t \quad 1]$ for p_2 and p_1 respectively.
   i. Find the matrix A associated with L. (4 marks)
   ii. If $p(t)=5t^2-3t+2$, compute $L[P(t)]$ directly and then by using **A.**(3 marks)

c). Let $A=\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and $S=\{e_1, e_2, e_3\}$ and $T=\{\overline{e_1}, \overline{e_2}\}$ be the natural bases for R^3 and R^2 respectively.

i. Find the unique linear transformation $L:R^3 \to R^2$ whose representation with respect to **S** and **T** is **A**. (5 marks)

ii. Let $S'=\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$ and $T'=\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right\}$ be ordered bases for R^3 and R^2, respectively. Determine the linear transformation $L:R^3 \to R^2$ whose representation with respect to S' and T' is **A**. (5 marks)

iii. Compute $L\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$, using L as determined in part (ii). (3 marks)

4. a). Let $L:P_2 \to P_2$ be a linear operator defined by $L(at^2+bt+c)=-bt-2c$. Find the corresponding matrix Eigen problem for each of the bases $S=\{1-t, 1+t, t^2\}$ and $T=\{t-1, 1, t^2\}$ for p_2. (10 marks)

b). Compute the Eigen values and associated Eigen vectors of

$$A=\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}.$$ (10 marks)

5. a). Let $L:R_3 \to R_3$ be defined by $L[(u_1 \quad u_2 \quad u_3)]=[2u_1-u_3 \quad u_1+u_2-u_3 \quad u_3]$. Let S be a natural basis for R_3. Find matrix representation of L with respect to S. Also let $S'=\{(1 \quad 0 \quad 1), (0 \quad 1 \quad 0), (1 \quad 1 \quad 0)\}$ be natural basis for R_3. Show that L is a diagonalizable linear transformation with respect to S'. (18 marks)
   b). Explain orthogonal matrix and give an example. (2 marks)

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