THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

MAT 107: LINEAR ALGEBRA I

Date: JULY 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

1. a). Given the system

$$\begin{cases} \frac{1}{y}+\frac{1}{x}-\frac{1}{z}=0 \\ \frac{2}{x}-\frac{2}{y}+\frac{1}{z}=3 \\ \frac{3}{x}-\frac{4}{y}+\frac{2}{z}=4 \end{cases}$$

i. Find the rank. (5 marks)

ii. State the nature of the system. (2 marks)

b). If a and b are any scalars, and **u** and **v** any vectors in **V, S**how that

i. $(a+b)u=au+bv$.(3 marks)

ii. $a(u+v)=au+av$. (3 marks)

iii. $u+o=u$. (3 marks)

c). Determine if $w=\{x,-10,z;x,z,\epsilon R\}$ is a subspace of R^3. (3 marks)

d). Let $v_1=\begin{pmatrix}2\\1\\1\end{pmatrix}$ and $v_2=\begin{pmatrix}1\\-1\\3\end{pmatrix}$. Determine whether the vector $v=\begin{pmatrix}1\\5\\-7\end{pmatrix}$ belongs to span $(v_1, \quad v_2)$. (5 marks) e).Let $L:R^3 \to R^3$ be defined by $L\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\begin{pmatrix}u_1+1\\2u_2\\u_3\end{pmatrix}$. Determine whether L is a linear transformation. (6 marks)

2. a) Let $R^3 \to R^3$ be defined by

$$L\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\begin{pmatrix}1&0&1\\2&1&2\\2&1&3\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$$

   i. Is L onto? (5 marks)
   ii. Find a basis for the range L and determine the dimension of the range. (5 marks)
   iii. Find Kernel L. (5 marks)
   iv. Is L one to one? (1 mark)

   b). Let $L:P_2 \to R$ be the linear transformation defined by

$L\left(a^2+bt+c\right)=\int_0^1\left(at^2+bt+c\right)dt$. Find $kerL$. (4 marks)

3. a). Let $v_1=\begin{bmatrix}1\\2\\1\end{bmatrix}, v_2=\begin{bmatrix}1\\0\\2\end{bmatrix}$ and $v_3=\begin{bmatrix}1\\1\\0\end{bmatrix}$. Show that vector $v=\begin{bmatrix}2\\1\\5\end{bmatrix}$ is a linear combination in R^3.
   (5 marks)
   b). In p_2, let $v_1=2t^2+t+2, v_2=t^2-2t, v_3=5t^2-5t+2$ and $v_4=-t^2-3t-2$. Determine whether the vectors $v=t^2+t+2$ belongs to span $\left[v_1, v_2, v_3, v_4\right]$.
   (5 marks)

   c). Are the vectors $v_1=\begin{bmatrix}1&0&1&2\end{bmatrix}, v_2=\begin{bmatrix}0&1&1&2\end{bmatrix}, v_3=\begin{bmatrix}1&1&1&3\end{bmatrix}$ in R^4 are linearly dependent or linearly independent? (5 marks)
   d). Determine whether the vectors
   $v_1=\begin{bmatrix}2&1\\0&1\end{bmatrix}, v_2=\begin{bmatrix}1&2\\1&0\end{bmatrix}$ and $v_3=\begin{bmatrix}0&-3\\-2&1\end{bmatrix}$ in M_{22} are linearly independent?
   (5 marks)
4. a).(i)Show that the set $S=\left\{t^2+1, t-1, 2t+2\right\}$ is a basis for the vector space p_2.
   (5 marks)
   (ii). Suppose $p_2=2t^2+6t+13$. Show that S is a basis for p_2.(5 marks)
   b). The set W of all $2\,by\,2$ matrices with trace equal to zero is a subspace of M_{22}. Show that the set $S=\left\{v_1, v_2, v_3\right\}$, where

$v_1=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, v_2=\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$ and $v_3=\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ is a basis for W. (6 marks)

c). Distinguish between the following

i. finite and infinite dimensional vector space. (2 marks)

ii. Natural basis and rank of a matrix. (2 marks)

5. a). Let $L:R_4 \rightarrow R_3$ be defined by
$L(u_1 \quad u_2 \quad u_3 \quad u_4)=[(u_1+u_2),(u_3+u_4),(u_1+u_3)]$. Find a basis for range L.
(5 marks)

b). Do the four vectors $v_1=\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, v_2=\begin{bmatrix}1\\1\\2\\0\end{bmatrix}, v_3=\begin{bmatrix}2\\-1\\0\\0\end{bmatrix}$ and $v_4=\begin{bmatrix}3\\1\\-1\\0\end{bmatrix}$ spans R^4 ?

(8 marks)

c). Find a basis for the solution space of the homogeneous system $(\xi I_3 - A)X=0$ for

$\xi=-2$ and $A=\begin{bmatrix}-3 & 0 & -1\\2 & 1 & 0\\0 & 0 & -2\end{bmatrix}$. (7 marks)

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