THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SPECIAL / SUPPLEMENTARY EXAMINATION

MAT 130: FOUNDATIONS OF APPLIED MATHEMATICS

Date: JULY 2019	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

QUESTION ONE (30 MARKS)

Given the vectors 2i +j-3k and i+5j+6k. Find

A vector perpendicular to both of them **(3marks)**

The angle between them. **(3marks)**

Show that the acceleration α of a particle which travels along a space curve with velocity v is given by $\alpha = \frac{dv}{dt}T + \frac{v^2}{\rho}N$ **(2marks)**

Convert the following numbers to octal

(i)1011010_2

(ii)1110011001100_2 **(4 marks)**

Verify $\sinh x + \cosh x = e^x$ **(4marks)**

Using the diagram explain the Kepler's three laws of planetary motion. **(4marks)**

Explain Newton's three laws of motion. . **(4marks)**

A particle moves so that its position vector is given by $r = \cos\omega t i + \sin\omega t j$ where ω is a constant. Show that

(i)the velocity **v** of the particle is perpendicular to **r** **(2marks)**

(ii) The accelerationα is directed towards the origin and has magnitude proportional to the distance from the origin **(2 marks)**

(iii)$r \times v = a\,constant\ vector$. **(2marks)**

QUESTION TWO (20marks)

Consider a car of mass M travelling round a circular track (corner) of radius r with a velocity v, whereas the centrifugal force tries to make the car fly away from the track (overturn) , the frictional forces often provide the necessary centripetal force to overcome the centrifugal force. Using a diagram show that the maximum safe speed is $v^2 \leq \frac{brg}{h}$

(12marks)

A ma of 80 kg climbs up a slop 400 m long inclined at 5^0 to the horizontal. Calculate the minimum work done by the man **(4 marks)**

Express $\sqrt{3-i}$ in polar form**.** **(4marks)**

QUESTION THREE (20marks)

(a) A projectile is launched with an initial speed of **u**(m/s) and at an angle θ to the horizontal . Determine

The time it takes to reach the highest point
The highest point reached
The time of flight back to earth
The range
Prove that the range of the projectile is a maximum when angle $\theta = 45^0$
Show that the path of a projectile is parabolic. **(12 marks)**

(b) Given that $z_1 = 1+3i\, z_2 = 2+5i\, z_3 = -3-4i$. Determine in terms of $a+bi$

$z_1 + z_2 - z_3$ **(2marks)**

$\frac{z_1 z_2}{z_1 + z_2}$ **(3marks)**

$z_1 z_2 z_3$ **(3marks)**

QUESTION FOUR (20MARKS)

Find the tangential acceleration and normal acceleration of a particle which moves on the ellipse $r = a\cos\omega t i + b\sin\omega t j$ **(8marks)**

Given$r_1 = 2i - j + k\, r_2 = i + 3j - 2k\, r_3 = -2i + j - 3k \wedge r_4 = 3i + 2j + 5k$. Find the scalars a, b and c such that $r_4 = a r_1 + b r_2 + c r_3$ **(4marks)**

A particle move along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$where t is the time.
Find the component of its velocity and acceleration at time $t = 1$in the direction $i - 3j + 2k$
(4marks)

Use the definition of the hyperbolic function $\cosh\theta$ and$\sinh\theta$ to show that $\cosh^2\theta-\sinh^2\theta=1$ **(4marks)**

QUESTION FIVE (20MARKS)

Given the space curve $x=t, y=t^2, z=\frac{2}{3}t^3$. Find

Unit vector T and curvature K. Hence determine the unit normal vector N **(12marks)**

Binormal vector B and the radius of torsion δ **(8marks)** ***END***