THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

MAY – JULY 2019 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

SCHOOL-FOCUSED

MAT 503: FLUID MECHANICS

Date: JULY 2019	**Duration: 3 Hours**
INSTRUCTIONS: Answer any THREE Questions	

Q1. a) Derive the Von. Karman's integral equation for steady flow under no pressure gradient. **(12 marks)**

b) For the velocity profile for laminar boundary layer

$$\frac{u}{U} = \sin \frac{\pi y}{2\delta}.$$

Find

i. an expression for the boundary layer thickness

ii. shear stress

iii. local drag coefficient

(8marks)

Q2. a) Discuss the Prandtl theory of boundary layer and its importance in fluid dynamics. **(8 marks)**

b). Derive the Prandtl boundary layer equations for the flow over a semi-infinite plate using the asymptotic approach.

(12 marks)

Q3. a) Find the velocity distribution and skin friction for unsteady flow of a viscous incompressible fluid over an oscillating plate.

(13 marks)

b) Show that for a two dimensionally axially symmetric boundary layer flow

i. $\int_0^\infty \left(1-\frac{u}{U}\right)^2 \frac{r}{a} dn = \delta_1 - \delta_2.$

ii. $\int_0^\infty \left(1-\frac{u}{U}\right)^3 \frac{r}{a} dn = \delta_1 - 3\delta_2 + \delta_3$

iii. $\int_0^\delta \left(\frac{u}{U}\right)^3 dy = \delta - \delta_1 - \delta_3$

where symbols have their usual meaning.

(10marks)

Q4. a) Consider the boundary layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + ¿.$$

with the boundary conditions $u=v=0\, at\ y=0$ and $u=U(x)\, as\ y \to \infty$, using similarity variables

$$\varphi = \frac{1}{C}\sqrt{U}, x^{\frac{m+1}{12}} f(n)$$

$$n = yC\sqrt{\frac{U}{x}} \wedge U = U_1 x^m.$$

where $U_1 \wedge C$ are constants.

Show that $f''' + ff'' + \beta(1 - f'^2) = 0$

where $\beta = 2m(m+1)$.

(23 marks)

Q5. b) Show that at a distance x from a leading edge of a flat plate parallel to a stream of unbounded fluid moving outside the boundary layer with velocity U, the tangential stress on the plate is

$$\frac{1}{4}\rho\left(\frac{v^3}{x}\right)^{\frac{1}{2}}\alpha$$

where $2\alpha^{\frac{-2}{3}} = \lim_{n\to\infty} F'(a)$ and $F(a)$ is the solution of the equation

$$f''' + ff'' = 0$$

for which $f(0) = f'(0) = 0$ and $f''(0) = 1$.

Show that the total drag D per unit breadth is given by

$$D = \frac{a\rho L V^2}{\sqrt{R_e}}$$

where $R_e = \frac{UL}{V}$.

(23 marks)

END