## THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A<br>MAIN EXAMINATION<br>MAY - JULY 2019 TRIMESTER<br>FACULTY OF COMMERCE<br>DEPARTMENT OF ACCOUNTING AND FINANCE<br>SPECIAL / SUPPMENETARY EXAMINATION<br>CMS 311: BUSINESS STATISTICS

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

## QUESTION ONE (30 MARKS) COMPULSARY

A population of 1,000 students spends an average of $\$ 10.50$ a day on dinner. The standard deviation of the expenditure is $\$ 3$. A simple random sample of 64 students is taken.
a) What are the expected value, standard deviation, and shape of the sampling distribution of the sample mean?
(10 MARKS)
b) What is the probability that these 64 students will spend a combined total of more than $\$ 715.21$ ?
(10 MARKS)
c) What is the probability that these 64 students will spend a combined total between $\$ 703.59$ and $\$ 728.45$ ?
(10 MARKS)

## QUESTION TWO (20 MARKS)

A. Two workers on the same job show the following results over a long period of time.


1. Which worker appears to be more consistent in the time he requires to complete the job? Explain.

## (7 MARKS)

2. Which worker appears to be faster in completing the job? Explain. (6 MARKS)
B. Suppose the manager of a plant is concerned with the total number of man-hours lost due to accidents for the past 12 months. The company statistician has reported the mean number of man-hours lost per month but did not keep a record of the total sum. Should the manager order the study repeated to obtain the desired information? Explain your answer clearly.
(7 MARKS)

## QUESTION THREE (20 MARKS)

A. The scores on an aptitude test required for entry into a certain job position have a mean of 500 and a standard deviation of 120 . If a random sample of 36 applicants has a mean of 546, is there evidence that their mean score is different from the mean that is expected from all applicants?
(10 MARKS)
B. The training department of a company wishes to determine if there is any difference in the performance between the workers that have completed a training program and those that have not completed the program. A sample of 100 trained workers reveals an average output of 74.3 parts per hour with a sample standard
deviation of 16 parts per hour. A sample of 100 who have not been trained has an average output of 69.7 parts per hour with a standard deviation of 18 parts per hour. Is there evidence of a difference in output between the two groups? Write a 95\% confidence interval estimate of the difference.
(10 MARKS)

## QUESTION FOUR (20 MARKS)

The price of the standard family saloon car and the company market share was recorded for a random sample of 12 car manufacturers.

| Selling price \$’00 | 137 | 138 | 125 | 142 | 168 | 145 | 135 | 145 | 160 | 146 | 136 | 160 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Market share \% | 14 | 15 | 10 | 8 | 9 | 7 | 11 | 5 | 3 | 5 | 7 | 2 |

Required:
a) Plot the data on a scatter diagram and comment.
b) Calculate the product moment correlation coefficient.
c) Interpret the result obtain in (b) above

## CMS 311 BUSINESS STATISTICS FORMULAE

## PARAMETERS

- Population mean $=\mu=\left(\Sigma X_{i}\right) / N$
- Population standard deviation $=\sigma=\operatorname{sqrt}\left[\Sigma\left(X_{i}-\mu\right)^{2} / N\right]$
- Population variance $=\sigma^{2}=\Sigma\left(X_{i}-\mu\right)^{2} / N$
- Variance of population proportion $=\sigma_{P}{ }^{2}=P Q / n$
- Standardized score $=Z=(X-\mu) / \sigma$


## Statistics

Unless otherwise noted, these formulas assume simple random sampling.

- Sample mean $=x=\left(\Sigma x_{i}\right) / n$
- Sample standard deviation $=s=\operatorname{sqrt}\left[\Sigma\left(x_{i}-x\right)^{2} /(n-1)\right]$
- Sample variance $=s^{2}=\Sigma\left(x_{i}-x\right)^{2} /(n-1)$
- Variance of sample proportion $=\mathrm{s}_{\mathrm{p}}{ }^{2}=\mathrm{pq} /(\mathrm{n}-1)$


## Counting

- $n$ factorial: $\mathrm{n}!=n *(n-1) *(n-2) * \ldots * 3 * 2 * 1$. By convention, $0!=1$.
- Permutations of $n$ things, taken $r$ at a time: ${ }_{n} P_{r}=n!/(n-r)$ !
- Combinations of $n$ things, taken $r$ at a time: ${ }_{n} \mathrm{C}_{r}=n!/ r!(n-r)!={ }_{n} \mathrm{P}_{r} / r$ !


## Probability

- Rule of addition: $P(A \cup B)=P(A)+P(B)-P(A \cap B)$
- Rule of multiplication: $P(A \cap B)=P(A) P(B \mid A)$
- Rule of subtraction: $P\left(A^{\prime}\right)=1-P(A)$


## Random Variables

In the following formulas, $X$ and $Y$ are random variables, and $a$ and $b$ are constants.

- Expected value of $X=E(X)=\mu_{x}=\Sigma\left[x_{i}{ }^{*} P\left(x_{i}\right)\right]$
- Variance of $X=\operatorname{Var}(X)=\sigma^{2}=\Sigma\left[x_{i}-E(x)\right]^{2}{ }^{*} P\left(x_{i}\right)=\Sigma\left[x_{i}-\mu_{\underline{x}}\right]^{2 *} P\left(x_{i}\right)$
- Normal random variable $=z$-score $=z=(X-\mu) / \sigma$
- Chi-square statistic $=X^{2}=\left[(n-1) * s^{2}\right] / \sigma^{2}$
- $\underline{\mathrm{f} \text { statistic }=f=\left[{s_{1}}^{2} / \underline{\sigma}_{1}{ }^{2}\right] /\left[{s_{2}}^{2} / \underline{\sigma}_{2}{ }^{2}\right]}$
- $\quad$ Expected value of sum of random variables $=E(X+Y)=E(X)+E(Y)$
- Expected value of difference between random variables $=E(X-Y)=E(X)-E(Y)$


## Sampling Distributions

- Mean of sampling distribution of the mean $=\mu_{x}=\mu$
- Mean of sampling distribution of the proportion $=\mu_{\mathrm{p}_{2}}=\mathrm{P}$
- Standard deviation of proportion $=\sigma_{p}=\operatorname{sqrt}[P$ * $(1-P) / n]=\operatorname{sqrt}(P Q / n)$
- Standard deviation of the mean $=\sigma_{\underline{x}}=\sigma / \operatorname{sqrt}(n)$
- Standard deviation of difference of sample means $=\sigma_{d}=\operatorname{sqrt}\left[\left(\sigma_{1}{ }^{2} / n_{1}\right)+\left(\sigma_{2}{ }^{2} / n_{2}\right)\right.$ ]


## Standard Error

- Standard error of proportion $=$ SE $_{p}=\mathrm{s}_{\mathrm{p}}=\operatorname{sqrt}\left[\mathrm{p}^{*}(1-\mathrm{p}) / \mathrm{n}\right]=\operatorname{sqrt}(\mathrm{pq} / \mathrm{n})$
- Standard error of difference for proportions $=\operatorname{SE}_{p}=\mathrm{s}_{\mathrm{p}}=\operatorname{sqrt\{ } \mathrm{p}^{*}(1-\mathrm{p})$ * $\left[\left(1 / n_{1}\right)\right.$ $\left.\left.+\left(1 / \mathrm{n}_{2}\right)\right]\right\}$
- Standard error of the mean $=$ SE $_{x}=s_{x}=s / \operatorname{sqrt}(n)$
- Standard error of difference of sample means $=$ SE $_{d}=\mathrm{s}_{\mathrm{d}}=\operatorname{sqrt}\left[\left(\mathrm{s}_{1}{ }^{2} / \mathrm{n}_{1}\right)+\left(\mathrm{s}_{2}{ }^{2} /\right.\right.$ $\underline{n}_{2}$ )]
- Standard error of difference of paired sample means $=\mathrm{SE}_{\mathrm{d}}=\mathrm{S}_{\mathrm{d}}=$ \{ sqrt $\left[\left(\sum\left(d_{i}-d\right)^{2} /(n-1)\right]\right\} / \operatorname{sqrt}(n)$


## Discrete Probability Distributions

- Binomial formula: $\mathrm{P}(\mathrm{X}=\mathrm{x})=\mathrm{b}(x ; n, P)={ }_{n} \underline{C}_{x}{ }^{*} \mathrm{P}^{\mathrm{x}}$ * $(1-\mathrm{P})^{\mathrm{n}-\mathrm{x}}={ }_{n} \underline{\mathrm{C}}_{x}{ }^{*} \mathrm{P}^{\mathrm{x}}{ }^{*} \mathrm{Q}^{\mathrm{n}-\mathrm{x}}$
- Mean of binomial distribution $=\mu_{x}=n$ * $P$
- Variance of binomial distribution $=\sigma_{x}^{2}=n^{*} P^{*}(1-P)$
- Negative Binomial formula: $P(X=x)=b^{*}(x ; r, P)={ }_{x-1} \underline{C}_{r-1}{ }^{*} P^{r} *(1-P)^{x-r}$
- Mean of negative binomial distribution $=\mu_{x}=r Q / P$
- Variance of negative binomial distribution $=\sigma_{x}{ }^{2}=r^{*} Q / P^{2}$
- Poisson formula: $\mathrm{P}(x ; \mu)=\left(e^{-\mu}\right)\left(\mu^{x}\right) / x$ !
- $\quad$ Mean of Poisson distribution $=\mu_{x}=\mu$
- Variance of Poisson distribution $=\sigma_{x}^{2}=\mu$
- Multinomial formula: $P=\left[n!/\left(n_{1}!{ }^{*} n_{2}!{ }^{*} \ldots n_{k}!\right)\right]^{*}\left(p_{1}{ }^{n_{1}}{ }^{*}{ }^{2} p_{2}{ }^{n_{2}}{ }^{*} \ldots{ }^{*} p_{k}{ }^{n} k\right)$


## CORRELATION AND REGRESSION

> Covariance $\left(\operatorname{cov}_{(\mathrm{xy})}\right.$ or $\left.\mathrm{S}_{\mathrm{xy}}\right)={ }^{\frac{1}{N}} \sum\left(\mathrm{x}_{\mathrm{i}}-\ddot{\mathrm{X}}\right)\left(\mathrm{y}_{\mathrm{i}}-\dot{\mathrm{Y}}\right)$
> $\operatorname{cov}_{(\mathrm{xy})}$ or $\mathrm{S}_{\mathrm{xy}}=\left({ }^{\frac{1}{N}} \sum \mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}\right)-\ddot{\mathrm{X}} \dot{\mathrm{Y}}$

Coefficient of Correlation $\left(\mathrm{r}_{\mathrm{x}}\right)=\frac{s_{x y}}{s_{x} s_{y}}$

$$
r=\frac{n \sum x y-\left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^{2}\right)-\left(\sum x\right)^{2}} \sqrt{n\left(\sum y^{2}\right)-\left(\sum y\right)^{2}}}
$$

Rank correlation coefficient or spearman's rank correlation coefficient ( $r_{\mathrm{s}}$ )
$\rho=1-\frac{6 \sum d_{i}^{2}}{n\left(n^{2}-1\right)}$
Kendall Rank correlation

$$
\tau=\frac{n_{c}-n_{d}}{\frac{1}{2} n(n-1)}
$$

Pearson $r$ correlation
$r=\frac{\mathrm{N} \sum \mathrm{xy}-\sum(\mathrm{z})(\mathrm{y})}{\sqrt{\left.N \sum x^{2}-\sum\left(x^{2}\right)\right]\left[N \sum y^{2}-\sum\left(y^{2}\right)\right]}}$
Method of least squares
$\sum y=n a+b \sum x_{i}$
$\sum y_{i} x_{i}=a \sum x_{i}+b \sum x_{i}^{2}$
$\sum \mathrm{x}_{\mathrm{i}}=\mathrm{na}+\mathrm{b} \sum \mathrm{y}_{\mathrm{i}}$
$\sum x_{i} y_{i}=a \sum y_{i}+b \sum y_{i}^{2}$
$b=\frac{n\left(\sum X Y\right)-\left(\sum X\right)\left(\sum Y\right)}{n\left(\sum X^{2}\right)-\left(\sum X\right)^{2}}$
$a=\frac{\left(\sum Y\right)-b\left(\sum X\right)}{n}$
LSMAA $=a+b X$
$\mathrm{b}=\mathrm{r}_{\mathrm{x}}-\frac{s y}{s x}$
$a=\dot{Y}-b \ddot{X}$
*END*

