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MAIN EXAMINATION

SEPTEMBER – DECEMBER 2020 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 465: TIME SERIES ANALYSIS

Date: APRIL 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Question 1(Compulsory)

a). i) Describe the components which make up a time series. **(8 marks)**

ii) Explain the objectives of a time series analysis. **(2 marks)**

b). Given that $X_t = e^{it\theta}$,show that the effect of the first difference filter, the series is magnified by $g(\theta) = 2sin\frac{\theta}{2}$ **(5marks)**

c). Let $X_t: t = 0, \mp 1, \mp 2, \ldots..$ be a stochastic process given by $X_t = a + bt + e_t$ where e_t is a sequence of independent random variables distributed with $\mu = 0, \wedge \sigma^2$ as $e_t N(0, \delta^2)$.Show that $X_t = (X_t - X_{t-1})$ is stationary **(5marks)**

d). Explain the following statements

i) Time series is stationary in the weak sense. **(3 marks)**

ii) Time series is stationary in the strict sense **(3marks)** e). Explain the situations in which one would employ the two main models of time series analysis **(4marks)**

Question 2

a). Given that $Y_t = Sin\theta t$, Show that it is a weak series and determine the covariance function of the series **(10 marks)**

b) Consider an AR(1) given by $\alpha X_t + e_t$, where e_t is a purely random process and $|\alpha| < 1$ and let $f \quad (\lambda)$ be the normalized spectrum of X_t

Show that $f \quad (\lambda = \frac{1-\alpha^2}{2\pi(1-2\alpha cos\lambda+\alpha^2)}$ **(10marks)**

Question 3

a). i) Define a moving average process of order q, [MA(q)] hence: **(2marks)**

ii) Determine its mean and variance**.** **(4marks)**

iii) Obtain its covariance and autocorrelation functions. **(4marks)**

iv) State giving reason, whether it is a second order stationary. **(2marks)**

b) .Consider the autoregressive process of order 2 given by $X_t = \propto_1 x_{t-1} + \propto_2 x_{t-2} + e_t$ where

(e_t) is a purely random process. Derive the Yule-Walker equation and hence its general solution. What are the values of $\propto_1$ and $\propto_2$ for which the process is stationary**.** **(8marks)**

Question 4

a).Given that $X_t = \sum_{k=0}^{\infty} \alpha^k e_{t-k}$ where e_t is a purely random process, obtain the autocorrelation function. **(10 marks)**

b).Let $X_t = x_{t-1} + e_t$ where the sequence e_t is a sequence of uncorrelated random variable with mean μ and variance δ^2 and $x_0 = 0$, Show that the time series in non Stationary **(10marks)**

Question 5

a) Consider two MA (1)processes P and Q given by

$$P: X_t = e_t + \lambda e_{t-1}$$

$$Q: X_t = e_t + \frac{1}{\lambda} e_{t-1}$$

Obtain the autocorrelation function for the two processes. Is a moving average process uniquely identified from a given autocorrelation function give reason. **(8 marks)**

b) Consider a finite MA process defined as

$X_t = \sum_{j=0}^{m} \beta_{i,} e_{t-j}$, where e_t is a purely random process and let $f_x(\lambda)$ be the spectral density function of X_t

Show that $f_x(\lambda = \frac{\delta^2}{2\pi(1+2\beta_1 cos\lambda+\beta_1{}^2)}$ **(12marks)**

END