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MAIN EXAMINATION

SEPTEMBER – DECEMBER 2020 TRIMESTER

SCHOOL OF SCIENCE

DEPARTMENT OF MATHEMATICS & ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 263: PROBABILITY & STATISTICS IV

Date: DECEMBER 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

QUESTION ONE (30MARKS)

a)The random variables X_1, X_2 and X_3 have the joint $tri-variate$ probability density function

$$f(x_1,x_2,x_3)=\begin{cases}3\lambda(x_1+x_2)x_3, & 0<x_1<1, 0<x_2<2, 0<x_3<3\\ 0 & elsewhere\end{cases}$$

i) Determine λ

ii) Find the marginal p.d.f of X_3

iii) Find $\Pr\left[X_1 \le \frac{1}{2}, X_2 > 1, 1 < X_3 < 2\right]$ (8 marks)

b) Let $X_1, X_2, ..., X_n$ be a random sample from a distribution that has a mean of μ and variance σ^2. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample. Find the mean and variance $\bar{X}$.

(5 marks)

c) The random vector $\underset{\sim}{X} = (X_1, X_2)$ has p.d.f

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$$f\left(x_1,x_2,x_3\right)=\begin{cases}\frac{1}{12}\left(1+x_2\right), & 0<x_1<3, 0<x_2<2\\ 0 & elsewhere\end{cases}$$

Find the moment generating function of $\underline{X}$ (5 marks)

d) i) Define the characteristic function of a random variable X (2marks)

ii) Find the characteristic function of X if the p.d.f of X is

$$f\left(x\right)=\begin{cases}e^{-(x-2)}, & x>2\\ 0 & elsewhere\end{cases}$$ (4 marks)

e) Consider a random variable with moment generating function $M(t)$. Show that the second derivative of $\ln\left[M(t)\right]$ evaluated at $t=0$ is the variance of the random variable. (6marks)

QUESTION TWO (20MARKS)

a) Suppose that X_1, X_2 and X_3 are independent random variables with $E(X_i)=0$ and $E(X_i^2)=1$ for $i=1,2,3$. Determine $E\left[(-4X_3+X_2)X_1^2\right]$ (6 marks)

b) Suppose that the random vector has the 3-variate distribution given by

$$f\left(x_1,x_2,x_3\right)=\frac{20!}{x_1!x_2!x_3!\left(20-x_1-x_2-x_3\right)!}\left(\frac{1}{10}\right)^{x_1}\left(\frac{1}{5}\right)^{x_2}\left(\frac{3}{10}\right)^{x_3}\left(\frac{2}{5}\right)^{20-x_1-x_2-x_3}$$

i. Obtain the moment generating function $\underline{X}$ (3 marks)

ii. Determine the marginal distribution of X_2 and X_3 (4 marks)

iii. The conditional mean and variance of X_1 given that $X_2=X_2$ and $X_3=X_3$ (7 marks)

QUESTION THREE (20MARKS)

a) Suppose that $\underline{X} = (X_1, X_2, X_3)$ has the 3-variate normal distribution with density $f(\underline{x}) = c\exp.\left(-\frac{1}{2}Q\right)$ where

$$Q = \frac{1}{17}\left(11x_1^2 + 7x_2^2 + 5x_3^2 - 6x_1x_2 - 4x_1x_3 - 2x_2x_3 + 2x_1 - 16x_2 - 22x_3 + 24\right).$$

Identify $\underline{\mu}$ and $\sum$. Hence find c. (8marks)

b) Suppose the random variable X has Poisson distribution

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,\ldots$$

Determine the characteristic function of X (6 marks)

c) Suppose that X_1, X_2, X_3 are three independent random variables each having p.d.f

$$f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, elsewhere \end{cases}$$

Let $Y = Max.(X_1, X_2, X_3)$. Determine the p.d.f of Y and hence compute $\Pr.(Y \le \frac{1}{2})$. (6 marks)

QUESTION FOUR (20MARKS)

a)The random variables X_1, X_2 and X_3 have the $tri-\text{var}iate$ probability density

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3} & 0 < x_1 < 1, 0 < x_2 < 1, x_3 > 0 \\ 0 & elsewhere \end{cases}$$

i) Find the marginal distribution of (X_1, X_3)

ii) Find the conditional probability density of (X_1, X_3) given $X_2 = \frac{1}{3}$ (7 marks)

b)Suppose that $X_1, X_2, ..., X_p$ are independent random variables. Show that

$$\Pr(a_1 < X_1 < b_1, a_2 < X_2 < b_2, ..., a_p < X_p < b_p) = \prod_{i=1}^{p}\Pr(a_i < X_i < b_i)$$ (5 marks)

c) Suppose that the random variables X_1, X_2 and X_3 are jointly distributed with means

$E(X_1)=E(X_2)=E(X_3)=0$ and variances $\sigma_1^2=3$, $\sigma_2^2=1$ and $\sigma_3^2=4$. Suppose the covariances are $\sigma_{12}=\sigma_{13}=\sigma_{23}=1$. If X_1 linearly regresses on X_2 and X_3, determine the linear regression coefficients. Hence find $E\left(X_1/X_2=2, X_3=7\right)$ (8 marks)

QUESTION FIVE (20MARKS)

a) Let $y=(y_1, y_2 y_3)'$ be a random vector with mean vector and covariance matrix

$$\mu=\begin{pmatrix}1\\-1\\3\end{pmatrix}, \quad \Sigma=\begin{pmatrix}1&1&0\\1&2&3\\0&3&10\end{pmatrix}$$

i. Let $z=2y_1-3y_2+y_3$. Find $E(z)$ and $Var(z)$

ii. $z_1=y_1+y_2+y_3$ and $z_2=3y_1+y_2-2y_3$. Find $E(z)$ and $\text{cov}(z)$ where $z=(z_1,z_2)'$. (10 marks)

b) The covariance matrix of $\underline{X}=\left(X_1, X_2, X_3\right)'$ is

$$\sum=\begin{bmatrix}4&-2&1\\-2&7&0\\1&0&5\end{bmatrix}$$

i) Find the total variation (2 marks)

ii) Find the generalized variation (2 marks)

iii) Determine the partial correlation coefficient between X_1 and X_2 given X_3 (6 marks)

END