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MAIN EXAMINATION

SEPTEMBER- DECEMBER 2020 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 301: ACTUARIAL MATHEMATICS II

Date: DECEMBER 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1 a) Define the meaning of competing risks **(2 marks)**

b) Explain the difference between a profit vector and a profit signature

(4 marks)

c) (i) Define the meaning of zeroisation in the context of unit linked policy.

(2 marks)

(ii) Explain why an insurance company might choose to zeroise the above profit vector. **(2 marks)**

d.) A 10-year unit-linked policy has the following profit vector:
(-50, -10, -10, 5, 5, 5, –3, 15, 40, 60)
Reserves are set up to zeroise future negative cash flows on the following basis:

Basis:

Mortality : The probability of death at each age is a constant 0.25% per annum
Rate of interest: 1.5% per annum
Determine the revised profit vector **(6 marks)**

e.) A company provides its employees with a benefit on disability before age 65. The benefit is a life annuity of 50% of salary at the date of disability.

(i) Draw and label a transition state diagram for this benefit. **(4 marks)**

(ii) Derive a formula for the expected present value of this benefit for a life aged *x* with a current annual salary of 20,000. **(4 marks)**

f.) Give the differences between multiple decrement lives and single decrement lives **(4 marks)**

Q2 a) The following is an extract of a decrement table assumed for a funeral plan, showing deaths (*d*) and withdrawals (*w*):

Age x	*(al)x d*	*(ad)x w*	*(ad)x*
85	10,000	1400	2,300
86	6,300	1,000	1,100
87	4,200		

It has been established that the independent rates of decrement of withdrawal are now only 50% of those assumed in the table above for the ages of 85 and 86. The underlying independent mortality rates are unchanged. Construct a revised decrement table to reflect this change. **(10 marks)**

b) (i) State the advantages and disadvantages of using crude mortality rates and directly standardised mortality rates as the comparison measure of mortality in two or more different populations **(4 marks)**

You are given the following data in respect of a sub-population:

Age	*Population*
50	100,000
55	95,000
60	80,000

Number of deaths in sub-population 1,250

(ii) Calculate the Standardised Mortality Ratio using ELT15 (Males) as the mortality rate for the standard population **(6 marks)**

Q3 a) A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i) Draw and label the appropriate transition diagram. **(5 marks)**

(ii) Set out an expression for the expected present value of this benefit. **(5 marks)**

b) A reversionary annuity is payable continuously beginning on the death of a life aged *x* to an annuitant aged *y*. Derive an expression for the present value of the reversionary annuity using random variables for the future lifetimes. **(5 marks)**

c.) State the features of unit-linked policies **(5 marks)**

Q4 A life insurance company issues a large number of 4-year unit-linked endowment assurance policies to lives aged 65 exact. Level premiums are payable annually in advance until maturity or earlier death. The company has performed a profit test on these policies and the profit vector per policy sold, ignoring surrenders, is as follows:

(185.21, –121.52, –5.28, 12.95)

(i) Calculate the profit signature per policy sold if negative non-unit fund cash flows are zeroised. **(6 marks)**

The company now wishes to allow for surrenders in its calculations. It assumes that at the end of the first and second policy years only, 3% of the surviving policyholders will surrender. Surrender values are equal to the bid value of units held (after deduction of the fund management charge) less a surrender penalty of 50.

(ii) Calculate the revised profit signature per policy sold after allowing for surrenders if negative non-unit cash flows are zeroised. **(12 marks)**

(iii) Calculate the net present value of the revised profit signature in part (ii), using a risk discount rate of 8% per annum. **(2 marks)**

Basis:

Mortality AM92 Ultimate

Interest earned on non-unit cash flows 5% per annum fund

Expenses Ignore

Q5 A life insurance company, is proposing to launch a “Low Start” unit-linked endowment policy for a term of 3 years under which premiums increase by a fixed monetary amount each year and are payable yearly in advance throughout the term of the policy or until earlier death. The premium payable and the amount of premium allocated to units in each policy year are as follows:

Policy Year	*Premium Payable*	*Allocation Rate*
	£	*%*
1	1500	50
2	2250	105
3	3000	115

If the policyholder dies during the term of the policy, the death benefit of £6,750 (i.e. the total amount of premiums due to be paid on the policy if held to maturity) or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of

each policy year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the policy year of exit.
The units are subject to a bid-offer spread of 6% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year. Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.
You should use the following assumptions in carrying out profit tests of this policy:

Rate of growth on assets in the unit fund	4.5% per annum
Rate of interest on non-unit fund cash flows	2.5% per annum
Mortality	90% AM92 Ultimate
Surrender	.5% of policies in force at the end of year 1 and 2.5% of policies in force at the end of year 2 then surrender
Initial expenses	£200
Renewal expenses	£55 per annum on the second and third premium dates
Initial commission	5% of first premium
Renewal commission	2.5% of the second- and third-years' premiums
Claim expense	£75 (payable only on death and surrender)
Risk discount rate	6.5% per annum

(i) Calculate the profit margin for the policy issued to a life aged 61 exact on the assumption that the company does not set up sterling reserves for this policy. **(13 marks)**
(ii) Explain why a life insurance company might need to set up non-unit reserves in respect of a unit-linked life assurance policy. **(3 marks)**
(iii) Calculate the profit margin for the policy on the assumption that the company does set up reserves for this policy. **(4 marks)**