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MAIN EXAMINATION

SEPTEMBER – DECEMBER 2020 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 503: NUMERICAL ANALYSIS I

Date: APRIL 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1 a) The number $\hat{x}$approximates $x$with a relative error of at most 10^{-N}.
What range of values can $\hat{x}$ given $x = 150$ and $N = 3$. (6 marks)

b) Use a polynomial of degree three to approximate $x = 0.35$ from the followings below: (7 marks)

x	0	0.1	0.3	0.4	0.5	0.6
f(x)	1.0	0.9	0.71	0.61	0.52	0.44

c) Define the following:

i) Absolute error (2 marks)
ii) Relative error (2 marks)
d) Use Newton divided difference scheme to approximate $P_{012}(2)$from table below:

x_i	0	1	3
f_i	1	3	2

(6 marks)

Q2 a) Prove the following: For arbitrary numbers $x_0 < x_1 < \cdots < x_m$ and there exists precisely one polynomial$P \in \prod, n = 1 = \sum_{i=0}^{m} n_i,$ which satisfies $P^k(x_i) = f_i^k, i = 0,1,\cdots,m, k = 0,1,\cdots,n_i - 1$ (10 marks)

b) Given $m = 1, n_0 = 2, n_1 = 3$determine the Hermite interpolating polynomial. (13 marks)

Q3 a) Use Neville Algorithm to determine $P_{0123}(x)$ given the following table: (13 marks)

x_i	0	1	3	5
f_i	1	3	2	4

b) Find the Taylor polynomial of degree 4 for lnxabout x*=1. Use the result to approximate the function at x=1.3 and estimate the error incurred and the actual value. (10 marks)

Q4 a) Calculate the integral

$$\int_0^1 t^5\, dt$$

by extrapolation over the steps $h_0 = 1, h_1 = 1/2, h_2 = 1/4$. (23 marks)

Q5 a) Sketch the quadrilateral bounded by the lines $x = 0, y = 0, y = 2(x - 1), y = \frac{1}{2}x + 1,$ and find the vertices of the quadrilateral. Find appropriate bilinear basis functions for interpolation on this quadrilateral, i.e., four bilinear functions Ɛ$_k$(x), k=0,1,2,3 which takes value 1 at vertex K and are zero at the other three vertices. Hence interpolate the function $e^{(-x-y)}$over this quadrilateral. First compute the function values at the vertices, secondly, construct the interpolation $\hat{f}(x,y)$ and finally find the value of this interplant at the point (1,1). Compare the result with the exact value of the original function f(x,y) at this point.

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