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MAIN EXAMINATION

SEPTEMBER– DECEMBER 2020 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 300: GROUP THEORY II

Date: DECEMBER 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1 a) What do you understand by an equivalence relation. (5 marks)

b) Show that conjugacy is an equivalence relation. (5 marks)

c) The elements of a dihydral group $D_3$is given by $D_3 = \{e, a, b, b^{2,} ab, ab^2\}$determine the conjugacy classes of

i) $[e]$ (2 marks)

ii) $[a]$ (3 marks)

iii) $[b]$ (3 marks)

d) Given $o(G) = 1084$illustrate the First Sylow Theorem. (5 marks)

e) Define a p-group and verify whether $D_4 = \{e, x, y, y^{2,} y^{3,} xy, xy^{2,} xy^3\}$is a p-group. (7 marks)

Q2 a) Given $D_3 = \{e, a, b, b^{2,} ab, ab^2\}$determine the following:

i) The Sylow 2-subgroup of D_3 (6 marks)

ii) The Sylow 3-subgroup of D_3 (4 marks)

b) Define the Centralizer of a in G. (3 marks)

c) Given $D_3 = \{e, a, b, b^2, ab, ab^2\}$ determine the Centralizer of the following:

i) $C_{D_3}(a)$ (2 marks)

ii) $C_{D_3}(b)$ (2 marks)

iii) $C_{D_3}(ab^2)$ (3 marks)

Q3 a) Within the context of a group G acting on a set S. what do you understand by

the orbit of s. (3 marks)

b) Given $S = \{1,2,3,4,5,6,7,8,9\}$and the group of permutations of G as

$G = \{e, (146), (164), (169), (196), (149), (194), (469), (496), (14)(69), (16)(49), (19)(46)\}$.

Determine the orbit of s when

i) $s = 1$ (3 marks)

ii)$s = 7$ (1mark)

iii)$s = 6$ (3 marks)

c) Define the Stabilizer of s, denoted by Stab(s) and show that the Stabilizer of s is a subgroup of G. (10 marks)

Q4 a) Prove the First Sylow theorem which states that if $p$is a prime and p^r divides the order of G, then G has a subgroup of order p^r. (10 marks)

b) Define the Center of a group. (2 marks)

c) Given the order of the group is given by the union of its conjugacy classes (i.e., conjugacy classes partition a group). Verify this using the Dyhydral group . (8 marks)

Q5 a) Prove that the number of of distinct Sylow p-subgroup is congruent to 1 modullo p,

I.e $1 + kp$. (20 marks)

END