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MAIN EXAMINATION

SEPTEMBER – DECEMBER 2020 SEMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 402: RISK MATHEMATICS

Date: DEC 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

QUESTION ONE

a) For a Poisson process with intensity λ, determine the probability that exactly one event will occur during a finite time interval of length t (3 marks)

b) Let S be a compound Poisson random variable with parameter $\lambda = 5$, $p(1) = \frac{1}{6}$ and $p(2) = \frac{5}{6}$. Use Panjer's recursion method to calculate the distribution of S (5 marks)

c) Let S be a compound Poisson random variable with parameter $\lambda = 7$ and uniform (0,1) distributed claims. Approximate $P(S < 10)$ using the central limit theorem approximation (4 marks)

d) An insurance company will be required to make a payout of \$600 on a particular risk event, which is likely to occur with a probability of 0.6. The utility for any level of wealth, w is given by: $U(w) = 3000 + 0.7w$.The insurer's initial level of wealth is \$ 5000. Calculate the minimum premium the insurer will require in order to take on the risk (4 marks)

e) Suppose that an insured party has an exponential utility function with parameter α.

i) What is the maximum premium P^{+} he is willing to pay for a risk X ? (3 marks)

ii) Suppose that the loss X is exponentially distributed with parameter $\theta = 0.02$ and that $\alpha = 0.0025$, what is the value of P^{+} (3 marks)

iii) Using the approximation formula, compute the approximated value of P^{+} (3 marks)

f) Define the following

i) Compound Poisson process

ii) Mixture distribution

iii) The Arrow-Pratt's measure of relative risk aversion

iv) The probability of ruin

v) Proportional reinsurance (5 marks)

QUESTION TWO

a) State and explain two limitations of utility theory (4 marks)

b) Assume that X and Y are independent standard normal random variables. Derive, showing each step the distribution of $Z = X + Y$? (4 marks)

c) An insurer knows from past experience that the number of claims received per month has a Poisson distribution with mean 10, and that claim amounts have an exponential distribution with mean 400. The insurer uses a security loading of 30%.

i) Calculate the insurer's adjustment coefficient (4 marks)

ii) Compute an upper bound for the insurer's probability of ruin, if the insurer sets aside an initial surplus of 1,200 (3 marks)

d) Consider a compound Poisson distribution with $\lambda = 5$ and $P(X = 1,2,3) = \frac{1}{4}, \frac{2}{3}, \frac{1}{12}$

Compute the distribution of S (5 marks)

QUESTION THREE

a) Let N be a random variable having the Binomial distribution with parameters $n = 20$ and $p = 0.6$. Let X_i be i.i.d random variables having the Exponential distribution with $\theta = 1$ and that X_i are independent of N. Suppose that $S = \sum_{i=1}^{N} X_i$

Calculate the following:

i) The expectation of S (2 marks)

ii) The variance of S (2 marks)

iii) The moment generating function of S (3 marks)

b) Assume there is a chance of 0.3 that there is a claim. When a claim occurs the loss is exponentially distributed with parameter $\theta = 2$. Suppose there are 400 independent policies with this loss distribution, compute the mean and variance of their aggregate loss (5 marks)

c) State and explain the net profit condition as applicable in ruin theory (3 marks)

d) State the Lundberg's inequality (2 mark)

e) Assume that X is exponentially distributed with parameter $\beta = 0.5$. Compute the value of the adjustment coefficient? (3 marks)

QUESTION FOUR

a) Stacy's utility function can be described $U(w) = \sqrt{w}$ .She faces a potential loss of $100,000 in the event that her should house burn down, which has a probability of 0.05.

   i) Calculate the maximum premium that Stacy would be prepared to pay to insure herself against the total loss of her house if her initial level of wealth was $150,000 and comment on your results (4 marks)

   ii) Suppose that Prudent Life plc has an initial wealth of $50 million and a utility function of the form $U(w) = w$, calculate the minimum premium Prudent Life plc would require in order to offer insurance to Stacy and comment on whether insurance is feasible in this instance. (3 marks)

b) If reported claims follow a Poisson process with rate 6 per day (and the insurer has a 24 hour hotline), calculate:

   i) The probability that there will be fewer than 3 claims reported on a given day (3 marks)

   ii) the probability that another claim will be reported during the next two-hour period (3 marks)

c) Derive the probability density of the sum of two independent random variables, each of which is gamma with parameter $\alpha = 1$ and $\beta = 1$? (4 marks)

d) Differentiate between a compound distribution and a mixed distribution (3 marks)

QUESTION FIVE

a) Prove that if $S_1, S_2, ..., S_m$ are independent compound Poisson random variables with Poisson parameters λ_i and claim distribution P_i for $i = 1, 2, ..., m$, then $S = S_1 + S_2 + ... + S_m$ is compound Poisson distributed with specifications $\lambda = \sum_{i=1}^{m} \lambda_i$ and $P(x) = \sum_{i=1}^{m} \frac{\lambda_i}{\lambda} P_i(x)$ (5marks)

b) Let S be a compound Poisson random variable with parameter $\lambda = 3$, $p(1) = \frac{7}{12}$ and $p(2) = \frac{5}{12}$. Use Panjer's recursion method to calculate the distribution of S (5 marks)

c) State and explain three axioms of the expected utility theory (6 marks)

d) Suppose that for $w < 10$, the insured's utility function is $U(w) = 10w - w^2$. What is the maximum premium P^+ as a function of $w, w \in [0,10]$ for an insurance policy against a loss 1 with probability 0.4? (4 marks)

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