**THE CATHOLIC UNIVERSITY OF EASTERN AFRICA**

**DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE**

**END OF SEP-DEC 2020 TRIMESTER EXAM**

**MAT 230: VECTOR ANALYSIS**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO TIME:2 HRS**

**QUESTION 1 (COMPULSORY)**

a) State which of the following are scalars and which are vectors giving reasons.
   i. Momentum
   ii. Energy
   iii. Speed
   iv. Weight

(4mks)

b) Find the projection of the vector $\hat{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\hat{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$ (5mks)

c) Find the angle between $\hat{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ $and$ $\hat{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ ( 5mks)

d) If $\hat{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\hat{B} = \sin t\hat{i} - \cos t\hat{j}$ find $\frac{d}{dt}(\hat{A} \times \hat{B})$ (6mks)

e) Given $\phi = 2x^3y^2z^4$ $find$ $\nabla.\nabla\phi$ (5mks)

f) If $\hat{A} = (3x + 6y)\hat{i} - 14yt\hat{j} + 20xz^2\hat{k}$ ,evaluate $\int \hat{A}.d\tilde{r}$ from $(0,0,0)$ $to$ $(1,1,1)$ along the path $x = t,\ y = t^2\ , z = t^3$ (5mks)

**QUESTION TWO 20MKS**

a) Prove
   i. $\nabla \times (\nabla\phi) = 0\,; Curl\,grad\,\phi = 0$ (7mks)

   ii. $\nabla.(\nabla \times A) = 0\,; (div\,curl\,A = 0)$ (7mks)

b) Find the unit tangent vector to any point on the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ ,hence or otherwise determine the unit tangent at the point where $t = 2$ (6mks)

**QUESTION THREE 20MKS**

a) Find the directional derivative of $\phi = (x^2 + y^2 + z^2)$ *at the point* $p(3,1,2)$ in the direction of the vector $yz\hat{i} + xz\hat{j} + xy\hat{k}$ (10mks)

b) A fluid motion is given by $\hat{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ ,show that the motion is irrotational and hence find the velocity potential. (10mks)

**QUESTION FOUR 20 MKS**

a) If $\hat{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$ ,evaluate $\iiint \hat{F} dv$ where $v$ is the region bounded by the surfaces $x = 0,\ y = 0,\ x = 2,\ y = 4,\ z = x^2,\ z = 2$ (10mks)

b) Use Greens theorem to evaluate $\int_c xydx + x^2dy \quad where$ $c$ is the boundary described counter clockwise of the triangle with vertices $(0,1)$ , $(1,0)$ , $(1,1)$ (10mks)

**QUESTION FIVE 20MKS**

Show that $\iint_s \hat{F}.\hat{n}ds = \frac{3}{2}$ where $\hat{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ *and* $s$ is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (20mks)

.....**End.....**