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MAIN EXAMINATION

SEPTEMBER-DECEMBER 2020 TRISEMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 330: ORDINARY DIFFERENTIAL EQUATIONS II

Date: DECEMBER 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

1. a) (i)State existence and uniqueness theorem for linear initial-value problems. (2 marks)

(ii) Show that the initial-value problem

$(x+1)y'' + 4y' = x^2 + 1, y(1) = 2, y'(1) = -5$ on interval $-\infty < x < \infty$ does

not satisfy the above theorem in (i) (2 marks)

(iii) Indicate an interval for which a unique solution will exist (2 marks)

b) Use the Wronskian to show that the functions e^x, e^{-x} and $sinhx$ are linearly dependent. (6 marks)

c) Show that $y_1 = x$ is a solution of $2x^2y'' + xy' - y = 0$ (3 Marks)

i. Use the method of reduction of order to find a second linearly independent solution of this differential equation and write the general solution (5 marks)

d) Find the order and degree of $e^x y'' - 3(y')^2 + 2xy = xe^x$ and write the differential equation in normal form (4 marks)

e) Distinguish between ordinary point and singular point (2 Marks)

f) At which point does the following differential equation have singular points

(i) $(1-x^2)y''-2xy'+6y=0$ (2 marks)

(ii) $(x^2+1)y''+6xy'-2y=0$ (2 marks)

2. Solve the differential equation $(1-x^2)y''-2xy'+12y=0$ by assuming a solution in power series form is valid near the origin (20 marks

3. a (i) Show that $y_1=\cos x$ and $y_2=\sin x$ are solutions of $y''+y=0$ (4 marks)

ii) Show that $y=c_1\cos x+c_2\sin x$ is also a solution where c_1 and c_2 are arbitrary constants (3 marks)

b (i) Show that $y_1=e^{-x}$ is a solution of $y''+3y'+2y=0$ (3 marks)

ii) Use the method of reduction of order to find a second linearly independent solution of this differential equation (8 marks)

iii) Write the general solution (2 marks)

4. a) Determine whether the following are linearly dependent or independent by using the Wronskian test

i) $1,5,x$ (5 marks)

ii) $\sin 3x, cos3x$ (5 marks)

b) Test for exactness and solve $(1+x^2)y''+4xy'+2y=sec^2x$ given that $y=0, y'=1$ when $x=0$ (10 marks)

5 a) Solve $y''-2\tan x\,y'+5y=0$ (10 marks)

b) Show that the functions $f(x)=x^2$ and $g(x)=\sum_{n=1}^{\infty}An\sin nx$ are orthogonal to each other in the in the interval $-\pi\le x\le\pi$, and then obtain a corresponding orthogonal set (10 marks)

END