THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

SEPTEMBER-DECEMBER 2020 TRISEMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 204: LINEAR ALGEBRA II

Date: DECEMBER 2020	**Duration: 2 Hours**
INSTRUCTIONS: Answer any THREE Questions	

1 a)Consider the functions $sin\ t\ and\ cos\ t$ in the vector space $C[-\pi,\pi]$ of continuous functions on the closed interval $[-\pi,\pi]$.$Show\ that$ $\sin t$ and $\cos t$ are orthogonal functions (5 marks)

b) Find the characteristic polynomial $\Delta(t))\ of\ A = \begin{bmatrix} 3 & -2 \\ 9 & -3 \end{bmatrix}$ (5 marks)

c) Find k so that $u = (1,-2,4k,3) and\ v = (3,2k,0,0)$ in $R^4$are orthogonal (5 marks)

d) Determine whether following matrices are positive definite

i) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ (3 marks)

ii) $\begin{bmatrix} 6 & -7 \\ -7 & 9 \end{bmatrix}$ (3 marks)

e) Consider vectors $u = (1,3,-6,4) and\ v = (3,5,1\ ,-2)$ in R^4 find

i. $\|u\|_\infty$ (2 marks

ii. $d_1(u,v)$ (3 marks)

f) Let V be the vector space of functions with basis $S = \{\text{sint}, \cos t, e^{3t}\}$ and let $D: V \to V$ be the differential operator defined by $D(f(t)) = \emptyset f(t)$ compute the matrix representing D in the basis s (4 marks)

2 a) Let V be the vector space of polynomial $f(t)$ with inner product

$\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$ Apply Gram-Schmidt or orthogonalization process to$\{1, t^{-1}, t^2, t^3\}$ to find an orthogonal basis $\{f_0, f_1, f_2, f_3\}$ with integers coefficients for $p_3(t)$ (10 marks)

b) Given vectors $u = (0, -8, 3)$ and $V = (-4, 3, -3)$ in R^3find

i. $\|v\|_1$ (3 marks)

ii. $d_\infty(u, v)$ (4 marks)

iii. $d_2(u, v)$ (3 marks)

3 a) Verify each of the following

i. Parallelogram law $\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2$

ii. Prove $\langle u, v\rangle = \frac{1}{4}[\|u + v\|^2 - \|u - v\|^2]$ (3 marks)

b) Find the matrix representation of each of the following linear operations F on R^3 relative to the usual basis $E\{e_1, e_2, e_3\}$ of R

i) F defined by $F(x, y, z) = (x + 2y - 3z, 4x - 5y - 6z, 7x + 8y + 9z)$ (4 marks)

ii) F defined by the 3×3 matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 5 & 5 \end{bmatrix}$ **(**2 marks)

iii) F defined by $F(e_1) = (1, 3, 5), F(e_2) = (2, 4, 6), F(e_3) = (7, 7, 7)$

(6 marks)

4 a) The vectors $u_1(1, 1, 0), u_2 = (1, 2, 3), u_3(1, 3, 5)$ form a basis S for Euclidean space R^3. Find the matrix A that represents the inner product in R^3 relative to the basis S (7 marks)

b) Let S consist of the following vectors inR^4 $u_1 = (1, 1, 0, -1), u_2 = (1, 2, 1, 3), u_3 = (1, 1 - 9, 2), u_4 = (16, -13, 1, 3)$

i. Show that S is orthogonal and a basis of R^4 (7 marks)

ii. Find the coordinates of an arbitrary vector $V = (a, b, c. d)$ in R^4 relative to the basis (6 marks)

5. a) Find the characteristic polynomial $\Delta(t)$ of each of the following matrices

i. A=$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{bmatrix}$ (8 marks)

ii. B=$\begin{bmatrix} 1 & 6 & -2 \\ -3 & 2 & 0 \\ 0 & 3 & -4 \end{bmatrix}$ (7 marks)

b) Let A= $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$, find all eigenvalues (5 marks)

END