THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

PART TIME PROGRAMME

MAT 542: STOCHASTIC PROCESSES

Date: DECEMBER 2018	Duration: 3 Hours
INSTRUCTIONS: Answer any THREE Questions	

Q.1 a) i) Define the generating function A(s) of a sequence $\{a_k\}$ **(2 marks)**

ii) Let X have the binomial distribution with parameters n and p thats is
$p_k = \binom{n}{k} p^k (1-p)^{n-k} \;\; k = 0,1,2, \ldots\ldots, n$
Obtain the probability generating function of X and hence obtain the mean and variance of X **(6 marks)**

b) Let $u_n = p(s_n = s_o)$ be the probability that a simple random walk revisits its starting point at time n. Show that $u_n = 0$, if n is odd, and $u_n = \binom{2m}{m} p^k q^m$ if n=2m is even(p is the probability of a rightward step and q is the probability of a leftwardstep) **(6marks)**

c) Consider two players playing a certain game. Player 1 has z amount and player 2 has $a - z$ amount so that the total amount used in the game is "a". After each game, a player either wins with probability p or loses with probability q . For player1, in the first trial if he wins then he has $z + 1$ and if he loses he has $z - 1$.Obtain the probability q_z that he will eventually lose all his money. Assume $p \neq q, p + q = 1$. **(9 marks)**

Q.2 a) Let X be random variable with geometric distribution $\{p_K\}$, where
$p_k = Prob\{X = k\} = q^k p, \; k = 0,1,2, \ldots\ldots \;\; 0 < q < 1$

Obtain:

i) The p.g.f of X

ii) The expected value of X, $E(X)$

iii) The variance of X, $var(X)$ **(8 marks)**

b) The random variable X has logarithmic series distribution if $p_k = Prob\{X = k\} = \frac{aq^k}{k}, k = 1,2, \ldots.\ \alpha = \frac{-1}{\log p}$
$0 < q = 1 - p < 1$.
Show that the p.g.f. of X can be expressed as
$P(s) = \frac{\log(1-sq)}{\log(1-q)}$. Hence or otherwise obtain $E(X)$ and $Var(X)$.

(9 marks)

c) Let X be a random variable assuming values 0,1,2,…. And let $P(X = j) = p_j$, and $(X > j) = q_j$, j=0,1,2,….. If $P(s)$ is the generating function of $\{p_j\}$ show that $Q(s) = \frac{1-P(s)}{1-s}$, $-1 < s < 1$ **(6marks)**

Q3. a) i) Explain clearly what is meant by a branching process.

ii) Give two examples of a branching process. **(5 marks)**

b) In a branching process , the nth generation of size Z_n depends on the $(n-1)th$ generation of size Z_{n-1}. The probability generation function of $\{Z_n\}$ is given as $\phi_n(s) = \phi_{n-1}(\phi(s))$

i) Show that $\phi_n(s) = \phi(\phi_{n-1}(s))$

ii) Find an expression of $\phi_n{}'(s)$ and $\phi_n{}''(s)$ **(6 marks)**

c) Assuming that Z_0=1 and postulating that $E(Z_1) = m$ and $var(Z_1) = E({Z_1}^2) - [E(Z_1)]^2 = \sigma^2$, where m and σ^2 exist and are finite, show that :

i) $E(Z_n) = m^n$

ii) $var(Z_n) = \sigma^2 m^{n-1} \frac{m^n - 1}{m-1}$ if $m \neq 1$ and $var(Z_n) = n\sigma^2$ if $m = 1$

(9 marks)

d) If the family-size distribution of a branching process has mass function $p_k = pq^k,\ k = 0,1,2, \ldots.\ 0 < p = 1 - q < 1$, show that the probability that the process becomes extinct ultimately is $\frac{p}{q}$ if $p \leq \frac{1}{2}$. **(4 marks)**

Q.4 a) Define each of the following examples of a stochastic process

i) A symmetric simple random walk

ii) A compound poisson process

iii) For each of the process in (i) and (ii), classify it as stochastic process according to its state space and the time that it operates on. **(4 marks)**

b) Consider the stochastic matrix

$P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ .Obtain P^n. **(14 marks)**

c) A random variable Y has a pgf $G_Y(s) = \frac{3+s}{6-2s}$, determine:

i) The mean **(2 marks)**

ii) The variance **(3 marks)**

Q5. a) Let $\phi(t)$ be the characteristic function of a random variable X. Show that if $\phi(t)$ is real it satisfies the inequality $1 - \phi(2t) \leq 4[1 - \phi(t)]$. **(6 marks)**

b) Let X be a symmetric random walk, that is , X and $-X$ have the same characteristic function. Show that $E(\sin t\, X) = 0$. **(6 marks)**

c) i) State the inversion theorem for characteristic functions.

ii) Find the probability density for which the characteristic function is

$$\phi(t) = \begin{cases} 1 - |t| \; for |t| \; \leq 1 \\ 0 \quad\quad for \; |t| > 1 \end{cases}$$ **(11 marks)**

END