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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

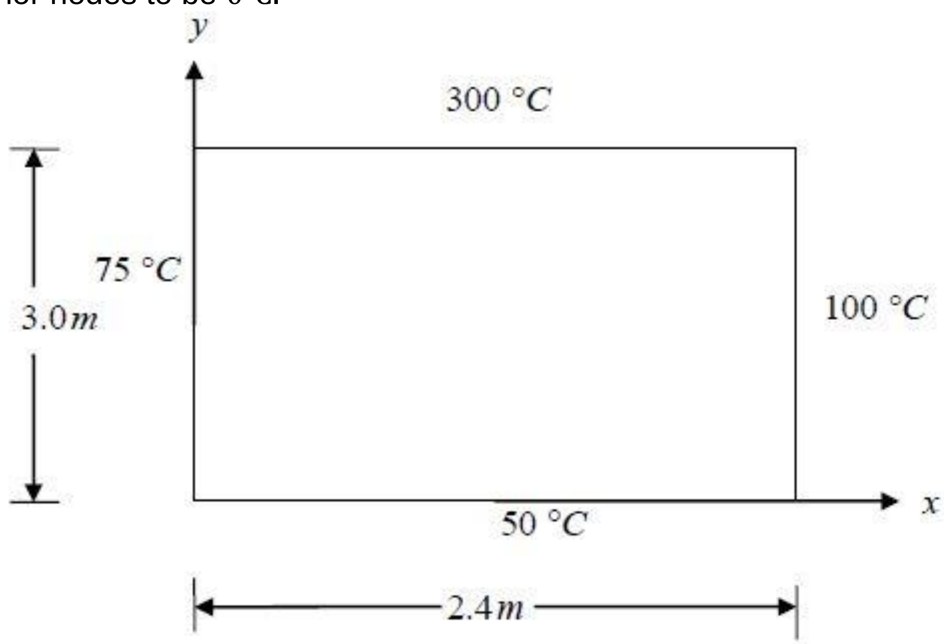
DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

PART TIME PROGRAMME

MAT 507: PARTIAL DIFFERENTIAL EQUATIONS

Date: DECEMBER 2018	**Duration: 3 Hours**
INSTRUCTIONS: Answer any THREE Questions	

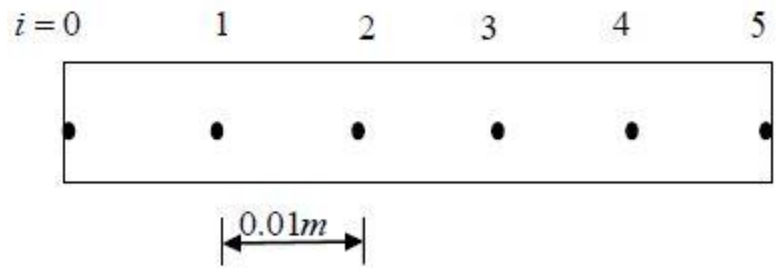
Q1. A plate $\mathbf{2.4}m\ x\mathbf{3.0}m$ is subjected to the temperature as shown in the figure below. Use a square grid of length of $\mathbf{0.6}m$. Assume the initial temperature at all interior nodes to be 0℃.



a) Using the **Gauss-Seidel method**, find the temperature at the interior nodes **8marks**

b) Find the maximum absolute relative error at the end of the second iteration **12 marks**

Q2. A rod of steel is subjected to a temperature of $\mathbf{100}$°C on the left end and $\mathbf{25}$°C on the right end. If the rod is of length $\mathbf{0.05m}$, use **Crank-Nicolson method** to find the temperature distribution in the rod from $\boldsymbol{t = 0}$ **to** $\boldsymbol{t = 6s}$. Use $\Delta \boldsymbol{x} = \mathbf{0.01m}, \Delta \boldsymbol{t} = \mathbf{3s}$. Below is the schematic diagram showing the node representation in the model. **20marks**



Given that, $\boldsymbol{k} = \mathbf{54}\frac{\boldsymbol{W}}{\boldsymbol{m-K}}, \ \boldsymbol{\rho} = \mathbf{7800}\frac{\boldsymbol{kg}}{\boldsymbol{m^3}}, \ \boldsymbol{C} = \mathbf{490}\frac{\boldsymbol{J}}{\boldsymbol{kg-K}}$.

Q3. a) Given that $\boldsymbol{U(x, y, z)} = \frac{\mathbf{1}}{\sqrt{x^2+y^2+z^2}}$, show that $\boldsymbol{U(x, y, z)}$ is a solution of the partial differential equation $\mathbf{U_{xx} + U_{yy} + U_{zz} = 0}$, for $(x, y, z) \neq (0,0,0)$. **12 marks**

b) State the following functions under Parabolic, Hyperbolic or Elliptic partial differential equations, giving reasons. **8marks**

i) $\boldsymbol{U(x, y)} = \frac{\partial^2 U}{\partial x^2} + 2\frac{\partial^2 U}{\partial x \partial y} - 3\frac{\partial^2 U}{\partial y^2} = \mathbf{0}$.

ii) $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \mathbf{0}$.

iii) $\frac{\partial T}{\partial t} = \boldsymbol{k}\frac{\partial^2 T}{\partial x^2}$.

iv) $\frac{\partial^2 y}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 y}{\partial t^2}$.

Q4. A second order partial differential equation is given by the relation;

$$\frac{\partial^2 U}{\partial x \partial y} = \boldsymbol{y} \cos \boldsymbol{x}.$$

a) Find the general solution of the equation **8 marks**

b) Find the particular solution which satisfies the condition $\boldsymbol{U(x, 0) = x}$ and $\boldsymbol{U(\pi, y) = y^2}$. **12 marks**

Q5. A rectangular plate of dimensions $\mathbf{2.4}\boldsymbol{m}\ \boldsymbol{x}\mathbf{3.0}\boldsymbol{m}$ is subjected to the temperatures as shown in the figure below. Using a square grid of length $\mathbf{0.6}\boldsymbol{m}$. Use the **Gauss-Seidel with successive over the relaxation method** with a weighting factor of $\mathbf{1.4}\boldsymbol{m}$ to find the temperature at the interior nodes. Conduct only the first iterations at all the interior nodes. Assume the initial temperature at all interior nodes to be $0°C$. **20marks**

END