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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR / PART TIME PROGRAMME

MAT 502: FLUID MECHANICS I

Date: DECEMBER 2018	**Duration: 3 Hours**
INSTRUCTIONS: Answer Question ONE and any other THREE Questions	

Q1. a) Discuss the Prandtl theory of boundary layer and its importance in fluid dynamics. **(8 marks)**

b) Derive the Prandtl boundary layer equations for the flow over a semi-infinite plate using the asymptotic approach. **(12 marks)**

Q2. a) Derive the Von. Karman's integral equation for steady flow under no pressure gradient. **(12 marks)**

b) For the velocity profile for laminar boundary layer
$\frac{u}{U} = sin\frac{\pi y}{2\delta}$.
find
i) an expression for the boundary layer thickness
ii) shear stress
iii) local drag coeffient **(8 marks)**

Q3. a) Find the velocity distribution and skin friction for unsteady flow of a viscous incompressible fluid over an oscillating plate. (10 marks)
b) Show that for a two dimensionally axially symmetric boundary layer flow

$i) \int_0^\infty \left(1-\frac{u}{U}\right)^2 \frac{r}{a} dn = \delta_1 - \delta_2.$

$ii) \int_0^\infty \left(1-\frac{u}{U}\right)^3 \frac{r}{a} dn = \delta_1 - 3\delta_2 + \delta_3$

$$iii)\int_0^{\delta}\left(\frac{u}{U}\right)^3 dy = \delta - \delta_1 - \delta_3$$

where symbols have their usual meaning.

Q4. Consider the boundary layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2}.$$

with the boundary conditions $u = v = 0\ at\ y = 0$ and $u = U(x)\ as\ y \to \infty$, using similarity variables

$$\varphi = \frac{1}{C}\sqrt{v\,U}, x^{\frac{m+1}{12}} f(n)$$

$n = yC\sqrt{\frac{U}{V\,x}}\ and\ U = U_1 x^m.$

where $U_1\ and\ C$ are constants.

Show that $f''' + ff'' + \beta\left(1 - f'^2\right) = 0$

where $\beta = 2m(m+1)$. **(20 marks)**

Q5. Show that at a distance x from a leading edge of a flat plate parallel to a stream of unbounded fluid moving outside the boundary layer with velocity U, the tangential stress on the plate is

$$\frac{1}{4}\rho\left(v\,\frac{v^3}{x}\right)^{\frac{1}{2}}\alpha$$

where $2\alpha^{\frac{-2}{3}} = \lim_{n\to\infty} F'(a\eta)$ and $F(a\eta)$ is the solution of the equation

$$f''' + ff'' = 0$$

for which $f(0) = f'(0) = 0$ and $f''(0) = 1$.

Show that the total drag D per unit breadth is given by

$$D = \frac{a\rho L V^2}{\sqrt{R_e}}$$

where $R_e = \frac{UL}{V}$. **(20 marks)**

END