THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR / PART TIME PROGRAMME

MAT 500: METHODS OF APPLIED MATHEMATICS

Date: DECEMBER 2018	Duration: 3 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) In Minkowiski space, define $x_1 = x$, $x_2 = y$, $x_3 = z$ and $x_0 = ct$. This is done so that the space time interval $dS^2 = {dx_0}^2 - {dx_1}^2 - {dx_2}^2 - {dx_3}^2$, where c is the velocity of light. Show that the metric in Minkowiski space is given by;

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$ **(8 marks)**

b) The components of a tensor A is equal to the corresponding components of a tensor B in one particular coordinate system. Show that $A = B$. **(7 marks**

c) Consider the Kernel of an integral equation to be defined as $K(x,t)$. Under what condition can the Kernel, $K(x,t)$ be referred to as a separate or degenerate Kernel? **(7 marks)**

d) Show that $y(x) = x + \frac{4x^2}{3}$ is a solution to the Fred Holm integral equation;$y(x) = x + \lambda \int_0^1 (x^2 t) y(t) dt$. **(8 marks)**

Q2. a) A Fred Holm integral equation of the second kind is given by;
$y(x) = x + \lambda \int_0^1 (xt^2 + x^2 t) y(t) dt$. **(12 marks)**
Solve for the solution of $y(x)$.

b) Use the anti-symmetry of ϵ_{ijk} to show that $\vec{A}.(\vec{A}X\vec{B}) = 0$. **(8 marks)**

Q3. a) Considering a Fred Holm integral equation with separable defined by the relation; $y(x) - \int_0^1 t(x-t)y(t)dt = 1$, for $0 \leq x \leq 1$. **(14 marks)**

b) Write $\nabla X(\nabla \phi)$ in ϵ_{ijk} notation so that it becomes zero (0). **(6 marks)**

Q4. a) An integral equation is given by; $y(x) = \frac{2x}{3} + \int_0^1 xty(t)dt$. Show that $y(x) = x$ is a solution to the integral equation. **(15 marks)**

b) Let $A_i, \ldots\ldots\ldots\ldots, n$. What will be the rank of its derivative? **(5 marks)**

Q5. a) Verify that $\cos 2x$ is a solution to the Fred Holm integral equation of the second kind; $y(x) = \cos x + 3\int_0^{\pi} K(x,t)y(t)dt,$ **(15 marks)**

Where, $K(x,t) = \begin{cases} \sin t \cos x, t < x \\ \sin x \cos t, t < \pi \end{cases}$

b) Show that the velocity of a fluid at any point is component of a contravariant vector. **(5 marks)**

END