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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 437: NUMERICAL ANALYSIS II

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Solve the following system using Gaussian elimination with pivoting

$$0.2x + 0.3y + 0.4z = 0.20$$
$$0.8x + 0.5y + 0{,}6z = 0.36$$
$$0.7x + 0.6y + 0.5z = 0.34$$

(8 marks)

b) With, $x^{(0)} = (1 \quad 1 \quad 1)$solve the system below to one decimal place using Jacobi's method. Perform the computations to three decimal.

$$10x_1 + 4x_2 + 5x_3 = 3.3$$
$$2x_1 + 20x_2 + 8x_3 = 6.6$$
$$5x_1 + 10x_2 + 20x_3 = 8.5$$

(9 marks)

c) Solve the equations

$$x + y + 3z = 6$$
$$2x + 5y + 9z = 15$$
$$4x + 5y + 15z = 29$$

by LU decomposition using the decomposition

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 5 & 9 \\ 4 & 5 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(8 marks)

d) The data below are known to obey a law of the form $y = a + bx$

(5 marks)

x	1	2	3	4	5
y	0.5	0.8	1.1	1.4	1.7

Find a and b

Q2. a) A least square line is given by $z = a + bx + cy$. Derive its normal equations

(5 marks)

b) Given the data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

Linearize the function $y = ax^b$ and find the constants a and b **(8 marks)**

c) Find the Tylor series solution of the differential equation $\frac{dy}{dx} x + y; y(0) = 1$

up to the term in x^5 **(7 marks)**

Q3. a) Use improved Euler method to find y at $x = 1$ in five steps given

$\frac{dy}{dx} x + y; y(0) = 1$

(14 marks)

b) Use Crammer's method to find the value of x,y and z in the following system

$2x + 3y + z = 1$

$3x - y + 2z = 8$ **(6 marks)**

$-4x + 3y + 3z = -1$

Q4. Use the fourth order Runge Kutta method to find y(1) to 5 decimal place given $\frac{dy}{dx} = 2x - y; y(0) = 1$ in 5 steps. Compare this solution with the analytical one

(20 marks)

Q5. a) Find the eigen values of $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and the corresponding eigen vectors

(5 marks)

b) Find the least square parabola $y = a + bz + cx^2$ that best fit the data

(5 marks)

x	-2	-1	0	1	2	3
y	-5	0	3	4	3	0

c) Find the LU decomposition of $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ such that the main diagonal entries of u are equal to 1. Hence solve the system of equations

$$x_1 + 3x_2 + 8x_3 = 4$$
$$x_1 + 4x_2 + 3x_3 = -2$$
$$x_1 + 3x_2 + 4x_3 = 1$$

(10 marks)

END