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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 364: DESIGNA ND ANALYSIS OF SAMPLE SURVEYS

Date: DECEMBER 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

QUESTION ONE

a) Show that for a simple random sample,

i) The sample mean $\bar{y}$, is an unbiased estimate of the population mean $\bar{Y}$.**(6 marks)**

ii) The estimate of the population total $\hat{Y}$, is an unbiased estimate of the population total Y . **(5 marks)**

iii) Show that the sample variance s^2 for a simple random sample is an unbiased estimate of the population variance S^2 . **(7marks)**

b) In a population with $N = 6$, the values of the observations $y_i\,(i = 1,2,34,5,6)$ are 8, 3, 1,11,4 and 7.

i) Calculate the sample mean $\bar{y}$ for all simple random samples of size 3, and hence verify that $\bar{y}$ is an unbiased estimate of $\bar{Y}$. **(5 marks)**

ii) Calculate the population total $\hat{Y}$ for all possible simple random samples of size 3, and verify the relation given by,

$E(\hat{Y}) = \frac{n}{N} Y$, where Y is the population total. **(3 marks)**

iii) Calculate s^2 for all simple random samples of size 3 from the same population and verify that $E\left(s^2\right) = S^2$. **(4 marks)**

QUESTION TWO

a) Show that the variance of the mean $\bar{y}$ from a simple random sample is,

$$V(\bar{y}) = \frac{S^2}{n}(1 - f)$$

Where f denotes the sampling fraction. **(10 marks)**

b) Evaluate the variance of the population total estimate $\hat{Y}$ from a simple random sample. **(4 marks)**

c) Signatures to a petition were collected on 676 sheets. Each sheet had enough space for 42 signatures. The numbers of signatures per sheet were counted on a random sample of 50 sheets with the results shown in the table.

Number of signatures (y_i)	Frequency (f_i)
42	23
41	4
36	1
32	1
29	1
27	2
23	1
19	1
16	2
15	2
14	1
11	1
10	1
9	1
7	1
6	3
5	2
4	1
3	1

Estimate the total number of signatures to the petition and the 95% confidence limits.

(6 marks)

QUESTION THREE

a) Show that if in every stratum from stratified random sampling, the sample estimate $\bar{y}_h$ is unbiased, then $\bar{y}_{st}$ is an unbiased estimate of the population mean $\bar{Y}$. **(5 marks)**

b) show that for stratified random sampling,

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h S_h^2 \frac{(1-f_h)}{n_h}.$$ **(7 marks)**

c) The results from a simple random sampling from a stratified population are given in the following table.

Stratum	N_h	n_h	$\bar{y}_h$	S_h^2
1	100	20	115.7	57.6
2	300	20	147.2	46.9
3	400	30	133.6	75.3

i) Evaluate $\bar{y}_{st}$. **(3 marks)**

ii) Find the standard error of the estimate $\bar{y}_{st}$. **(4 marks)**

QUESTION FOUR

a) Show that with proportional allocation from stratified random sampling,

$$V(\bar{y}_{st}) = \frac{1-f}{n} \sum_{h=1}^{L} W_h S_h^2.$$ **(5 marks)**

b) Let V_{prop} denote the variance for the estimate of the population mean per unit in stratified random sampling under proportional allocation, and let V_{ran} denote the variance of the estimate of the population mean in a simple random sample. Show that $V_{prop} \le V_{ran}$.

(8 marks)

c) In a population with $N = 6$ and $L = 2$, the values of y_{hi} are 0, 1, 2 in stratum 1 and 4,6,11 in stratum 2. A sample of $n = 4$ is to be taken. Show that the optimum allocation n_h when rounded to integers are $n_h = 1$ in stratum 1 and $n_h = 3$ in stratum 2. **(4 marks)**

d) Show that in systematic random sampling $\bar{y}_{sy}$ is an unbiased estimate of the population mean given by $\bar{Y}$. **(3 marks)**

QUESTION FIVE

a) If variates y_i, x_i are measured on each unit of a simple random sample of size n, assumed large, and $\hat{R} = \frac{\bar{y}}{\bar{x}}$ is the ratio estimate of the population means, show that $\hat{R}$ is an unbiased estimate of the ratio of the population means $R = \frac{\bar{\bar{Y}}}{\bar{\bar{X}}}$. **(5 marks)**

b) Show that the sample proportion $p = a/n$ from a simple random sample is an unbiased estimate of the population proportion $P = A/N$, where a is the sample total, A is the population total, n is the sample size and N is the population size. **(3 marks)**

c) Show that

$$V(\hat{A}) = \frac{N^2 PQ}{n}\left(\frac{N-n}{N-1}\right).$$

(5 marks)

d) For a population with $N = 6$, $A = 4$, $A' = 2$, work out the value of a for all possible simple random samples of size 3, and verify that;

i) $E(p) = P.$ **(4 marks)**

ii) $\frac{N-n}{(n-1)N}pq$ Is an unbiased estimate of the variance of p .**(3 marks)**

END