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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 361: TESTS OF HYPOTHESIS I

Date: DECEMBER 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define the following terms as used in hypothesis

i) Type I error
ii) Type II error
iii) The critical region
iv) The power of a test **(3 marks)**

b) An experimentis performed four times where the probability of success is θ.The hypothesis to be tested is such that $H_0: \theta = 0.6$ against $H_1: \theta = 0.25$ where the null hypothesis is rejected if the number of successes is less than 3. Determine:

i) Probability of type I error **(2 marks)**
ii) Probability of type II error **(2marks)**
iii) The power of the test when $\theta = \frac{1}{7}$ **(2marks)**

c) i) State the conditions under which t-test is used in testing hypothesis about the mean. **(3 marks)**

ii) A teacher administered a CAT to his students whose total marks were 30. He suspected that the mean mark should be 12 marks. The marks of 10 of his students were:

11 14 13 12 13 12 13 11 12 14

Is there a reason to believe that the teacher is right at $\alpha = 0.05$ level of significance? **(5 marks)**

d) Suppose X is a single observation from a population with probability density function given by

$$f(x) = \begin{cases} \theta x^{\theta-1}, 0 < x < 1 \\ 0 \quad , elsewhere \end{cases}$$

Determine the most powerful test, with significance level $\alpha = 0.05$ for testing the hypothesis

$H_0: \theta = 3$ against $H_1: \theta = 2$ **(6 marks)**

e) Let X be a single observation from a population with probability density function

$$f(x, \theta) = \begin{cases} \frac{\theta^x e^{-\theta}}{x!} & for\ x = 0,1,2 \dots \dots \\ 0, elsewhere \end{cases}$$

where $\theta \geq 0$.

Find the likelihood ratio critical region for testing the hypothesis

$H_0: \theta = 2$ against $H_1: \theta \neq 2$ **(6 marks)**

Q2. a) Let X be normally distributed with mean μ (unknown) and variance σ^2 (known). Determine the critical region for testing $H_0: \mu = \mu_0$ against $H_1: \mu < \mu_0$ at α level of significance. **(10 marks)**

b) The values 21,22,18,27,32,25,24,19,26,33,29,19 and 28 were observed from a normal population whose variance is 27. Test the hypothesis $H_0: \mu = 27$ against $H_1: \mu < 27$ at $\alpha = 0.05$ level of significance. **(10 marks)**

Q3. a) Suppose that X is normally distributed with mean μ and unknown variance σ^2. Derive the critical region for testing the hypothesis $H_0: \sigma^2 = {\sigma_0}^2$ vs $H_1: \sigma^2 > {\sigma_0}^2$ at α level of significance if a random sample of size n is taken from the population. **(10 marks)**

b) The amount of a chemical thought to be a causative factor to a skin disease on some laboratory guinea pigs, is measured for a randomly selected sample of guinea pigs in mgm/cm^3 with the following results: 5.21,5.24,5.87,6.11,5.55,5,48,5.38,5.29,5.25,5.69,5.42,5.63,5.49,5.52,and 5.60.

If the mean for guinea pigs with the disease is known to be 5.50, test the hypothesis $H_0: \sigma = 0.3$ vs $H_1: \sigma > 0.3$ at $\alpha = 0.05$ level of significance.

(9 marks)

Q4. a) Let $x_1, x_2, \dots \dots, x_m$ be a random sample from a normal population with mean μ_1 and variance ${\sigma_1}^2$. Let $y_1, y_2, \dots \dots, y_n$ be another sample from a normal population with mean μ_2 and variance ${\sigma_2}^2$. Suppose that ${\sigma_1}^2 = {\sigma_2}^2 = \sigma^2$. Show that the likelihood ratio test for $H_0: \mu_1 = \mu_2 = \mu$ against $H_1: \mu_1 \neq \mu_2$ can be based on the critical region $\omega = \{\underline{x}, \underline{y}, : t > c\}$

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\left[\sum_{i=1}^{m}(x_i-\bar{x})^2+\sum_{i=1}^{n}(y_i-\bar{y})^2\right](\frac{1}{m}+\frac{1}{n})}{m+n-2}}}$$

where$\underline{x}$=$(x_1, x_2, \dots \dots ., x_m)$, $\underline{y} = (y_1, y_2, \dots \dots ., y_n)$, $\bar{x}$ and $\bar{y}$ are the respective sample means. **(13 marks)**

b) Two samples of a newly discovered insect are taken from two different populations living in two different regions of rainforest . The weights of the first sample in milligrams are 12,33,24,27,16,29,35,30,21,17 and 25. The weights of the second sample are 18,23,14,16,18,19,21,25,22,23,17 and 20. Test the hypothesis that the two populations have the same mean at $\alpha = 0.05$, assuming that the populations are normal with a common variance. **(7 marks)**

Q5. a) Let $x_1, x_2, \dots \dots ., x_m$ be a random sample from a normal population with mean μ_1 and variance $\sigma_1{}^2$. Let $y_1, y_2, \dots \dots ., y_n$ be another sample from a normal population with mean μ_2 and variance $\sigma_2{}^2$. Develop the test for the hypothesis $H_0: \sigma_1{}^2 = \sigma_2{}^2$ against $H_1: \sigma_1{}^2 \neq \sigma_2{}^2$ **(13marks)**

b) Njiris High School and Thika High School had a Mathematics competition where Njiris presented 8 students and Thika 11 students. Their scores were as follows:

Njiris: 73, 62, 41, 55, 69, 83, 47, 49
Thika: 68, 77, 40, 58, 79, 43, 89, 52, 69, 60, 39

Test the hypothesis that the variance in both groups is equal at $\alpha = 0.05$.
(7marks)

END