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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 330: ORDINARY DIFFERENTIAL EQUATIONS II

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Explain the difference between an IVP and a BVP. **3 marks**

b) Express the differential equation $\frac{d^4x}{dt^4}-2\frac{d^3x}{dt^3}+5\frac{d^2x}{dt^2}+3\frac{dx}{dt}-8x=6\sin 4t$ as a system of first order differential equations. **4 marks**

c) Use the Wronskian to determine whether each of the following functions are linearly independent or not:-

i) $f_1(x)=x, \quad f_2(x)=x^2, \quad f_3(x)=4x-3x^2$ **4 marks**

ii) $f_1(x)=1+x, \quad f_2(x)=x, \quad f_3(x)=x^2$ **4 marks**

d) Let $x^2y"-7xy'+16y=0.$

i) Show that $y=x^4$ is a solution to the equation **4 marks**

ii) Use reduction of order to find the second solution. **5 marks**

e) Determine the singular points of the differential equation $2x(x-2)^2y''+3xy'+(x-2)y=0$ and classify them as regular or irregular. **6 marks**

Q2. a) The equation $x^2y'' + xy' + (x^2 - 1)y = 0$ is called a Bessel's equation.

i) Identify the order of this Bessel equation. **1 mark**

ii) Show that $x = 0$ is a regular singular point. **3 marks**

iii) Assuming a Frobenius type solution, find the indicial roots **5 marks**

iv) Find the solution of the equation for the different indicial roots obtained above. **9 marks**

v) Hence (or otherwise), find the general solution of the equation. **2 marks**

Q3. a) Solve the system $\begin{cases} \frac{dx}{dt} - 3x - 6y = t^2 \\ \frac{dy}{dt} + \frac{dx}{dt} - 3y = e^t \end{cases}$ **10 marks**

b) Given that $y(1) = 2$, find $y(0.5)$ for $y' = x^2 + y^2$ using the modified Euler method with $n = 10$. **10 marks**

Q4. Let $x^2y'' + xy' + (x^2 - 1)y = 0$.

a) Show that $x = 0$ is a regular singular point. **3 marks**

b) Assuming a Frobenius type solution, find the indicial roots **6 marks**

c) Find the solution of the equation for the different indicial roots obtained above. **9 marks**

d) Hence (or otherwise), find the general solution of the equation. **2 marks**

Q5. The equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ is known as Legendre's equation of order *n.*

a) Find all the singularities of the equation. **2 marks**

b) Determine the type (regular or irregular) of singularities obtained above. **2 marks**

c) Find a series solution of the equation at $x = 0$ **12 marks**

d) Determine the first two Legendre polynomials from your solution. **4 marks**

END