



# THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

**A. M. E. C. E. A**

**MAIN EXAMINATION**

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**AUGUST – DECEMBER 2018 TRIMESTER**

**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE**

**REGULAR PROGRAMME**

**MAT 161: PROBABILITY AND STATISTICS II**

**Date: DECEMBER 2018**

**Duration: 2 Hours**

**INSTRUCTIONS: Answer Question ONE and any other TWO Questions**

- Q1. i) For two events  $A$  and  $B$ ,  $P(A) = \frac{3}{5}$ ,  $P(B) = \frac{3}{4}$  and  $P(A \cup B) = \frac{19}{20}$
- i) Determine  $P(A \cap B)$  **(2 marks)**
  - ii) Determine  $P(\sim A / B)$  **(3 marks)**
  - iii) Are  $A$  and  $B$  independent events? **(1 mark)**
- ii) An insurance company randomly selects three new employees randomly from a total of 10 applicants, six men and four women. Let  $X$  be the number of women hired.
- i) State the distribution of  $X$  **(1 mark)**
  - ii) Find the mean and standard deviation of  $X$  **(4 marks)**
  - iii) Find the probability that no women are hired **(2 marks)**
- iii) In a binomial distribution, the mean number of successes was 3 and the variance of the number of successes was 2.7. determine the probability of success and the number of trials **(4 marks)**
- iv) For a set of nine numbers,  $\sum(x - \bar{x})^2 = 60$  and  $\sum x^2 = 285$ . Find the mean and standard deviation of the numbers. **(5 marks)**
- v) Define the following terms
- i) Mutually exclusive events **(2 marks)**
  - ii) Exhaustive events **(2 marks)**
  - iii) Independent events **(2 marks)**

iv) Ordinary least squares estimator **(2 marks)**

Q2. a) A student is likely to wake up on time with probability  $\frac{1}{3}$ . If he wakes up on time, there is a probability of  $\frac{9}{10}$  that he will arrive in the dining hall in time for breakfast. If he oversleeps there is a probability of  $\frac{1}{2}$  that he will miss breakfast but on any occasion he arrives on time, he has breakfast. What is the probability that on any day he will miss breakfast? **(10 marks)**

b) The probability distribution of a random variable  $X$  is listed in the table below.

$E(X) = 1.5$  and  $Var(X) = 1.5$  Determine the values of the unknown probabilities **(10 marks)**

$X$	0	1	2	3	4
$P(X = x)$	$a$	$b$	$a + b$	$c$	$c$

Q3. a) The random variable  $X$  is best described by a uniform distribution with mean 50 and variance 25. Write down the probability distribution of  $X$  and hence calculate  $P(X > 50)$  **(6 marks)**

b) The number of accidents per week in a certain factory follows a Poisson distribution with variance 3.2. find the probability that

i) At least 6 but no more than 9 accidents occur per week **(3 marks)**

ii) Exactly 7 accidents occur in a fortnight **(4 marks)**

iii) More than 4 accidents occur in a particular week **(3 marks)**

c) Let  $X$  be a discrete random variable with probability distribution

$$P(X = x) = \begin{cases} \frac{x}{10} & \text{for } x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

Compute  $E(5X^3 - 2X^2)$  **(4 marks)**

Q4. a) A discrete random variable  $X$  has the following probability distribution

$X$	1	3	6	$n$	12
$P(X = x)$	0.1	0.3	$k$	0.25	0.15

i) Find the value of  $k$

ii) Given  $E(X) = 6$  find the value of  $n$

iii) Calculate the variance of  $X$

**(12 marks)**

- b) The moment generating function of a random variable  $X$  is given by  

$$m(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$$
 Find the mean, variance and probability density function of  $X$  **(8 marks)**

Q5. Consider the following data where  $x$  represents female life expectancy and  $y$  represents the income.

Country	Afghanistan	Sri Lanka	Bhutan	India	Pakistan	Bangladesh
$x$	42	50	47	58	57	73
$y$	143	179	197	335	384	423

- i) Find the equation of a suitable line of regression for this data by use of regression coefficients **(6 marks)**
- ii) Estimate the value of  $x$  for Nepal where the value of  $y = 160$  **(2 marks)**
- iii) Estimate the value of  $x$  for North Korea where the value of  $y = 858$  **(2 marks)**
- iv) Calculate the regression coefficient and interpret it **(10 marks)**

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