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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 301: ACTUARIAL MATHEMATICS II

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define the meaning of competing risks **(2 marks)**

b) Explain the difference between a profit vector and a profit signature. **(4 marks)**

c) i) In the context of with-profit policies, describe the super compound method ofadding bonuses. **(2 marks)**

ii) Suggest a reason why a life insurance company might use the super compound method of adding bonuses as opposed to the compound method. **(2 marks)**

d) The profit signature of a 3-year assurance contract issued to a life aged 57 exact; with a premium payable at the start of each year of Kshs 500 is (−250, 150, 200).
Calculate the profit margin of the contract.
Basis:
Mortality: AM92 Ultimate
Lapses: None
Risk discount rate: 12% per annum **(6 marks)**

e) Why study actuarial mathematics in higher education? **(4 marks)**

e) A certain population is subject to three modes of decrement: α, β and γ.

i) Write down an expression for $(aq)_x{}^{\alpha}$ in terms of the single decrement table probabilities $q_x{}^{\alpha}$, $q_x{}^{\beta}$, and $q_x{}^{\gamma}$, assuming each of the three modes of decrement is uniformly distributed over the year of age *x* to *x* + 1 in the corresponding single decrement table **(2 marks)**

ii) Suppose now that in the single decrement table α , $tq_x{}^{\alpha}=1-t^2q_x{}^{\alpha}$, $(0 \le t \le 1)$, while decrements β and γ remain uniformly distributed. Derive a revised expression for $(aq)_x{}^{\alpha}$ in terms of the single decrement table probabilities $q_x{}^{\alpha}$, $q_x{}^{\beta}$, and $q_x{}^{\gamma}$ **(6 marks)**

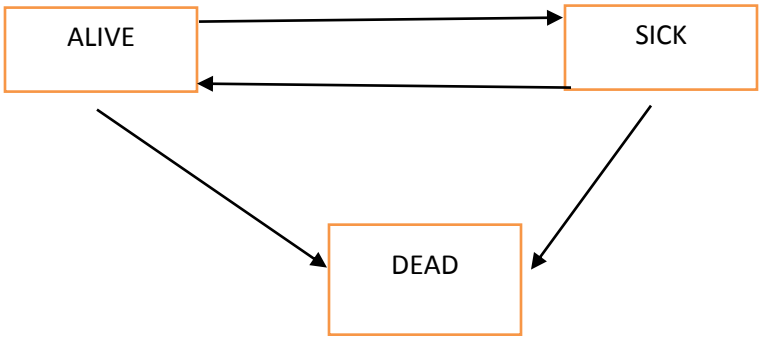
Q2. A life insurance company issues a 3-year unit-linked endowment assurance contract to a female life aged 60 exact under which level premiums of Kshs 5,000 per annum are payable in advance. In the first year, 85% of the premium is allocated to units and 104% in the second and third years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.75% of the bid value of the units is deducted at the end of each year.
If the policy holder dies during the term of the policy, the death benefit of Kshs 20,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On survival to the end of the term, the bid value of the units is payable.
The company holds unit reserves equal to the full bid value of the units but does not set up non-unit reserves. It uses the following assumptions in carrying out profit tests of this contract:

Mortality: AM92 Ultimate
Surrenders: None
Expenses: Initial: Kshs 600
Renewal: Kshs 100 at the start of each of the second and third policy years
Unit fund growth rate: 6% per annum
Non-unit fund interest rate: 4% per annum
Risk discount rate: 10% per annum

i) Calculate the expected net present value of the profit on this contract. **(18 marks)**

ii) State, with a reason, what the effect would be on the profit if the insurance company did hold non-unit reserves to zeroise negative cashflows, assuming it used a discount rate of 4% per annum for calculating those reserves. (You do not need to perform any further calculations.) **(2 marks)**

Q3. a) A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with μ= 2%, v= 4%,σ = 5% and ρ = 1%.
Calculate the present value of a sickness benefit of Kshs 2,000 p.a. paid continuously to a life now aged 40 exact and sick, during this period of sickness, discounted at 4% p.a. and payable to a maximum age of 60 exact. **(10 marks)**

b) A member of a pension scheme is aged 55 exact, and joined the scheme at age 35 exact. She earned a salary of Kshs 40,000 in the 12 months preceding the scheme valuation date.The scheme provides a pension on retirement for any reason of 1/80th of final pensionable salary for each year of service, with fractions counting proportionately.
Final pensionable salary is defined as the average salary over the three years prior to retirement.
Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables:

i) Calculate the expected present value now of this member s total pension. **(6 marks)**

ii) Calculate the contribution rate required, as a percentage of salary, to fund the future service element of the pension. **(4 marks)**

Q4. A five-year unit-linked policy issued to a life aged 50 exact has the following pattern of end of year cashflows per policy in force at the start of each year:

(-95.21, -30.18, -20.15, 77.15, 120.29)

i) Explain why a life office might need to set up non-unit reserves in respect of a unit-linked life assurance policy. **(4 marks)**

ii) Calculate the non-unit reserves required for the policy in order to zeroise negative cashflows assuming AM92 Ultimate mortality and those reserves earn interest at the rate of 5% per annum. **(4 marks)**

iii) Determine the net present value of the profits before and after zeroisation assuming the risk discount rate used is 8% per annum and state with reasons which of these figures you would expect to be higher. **(12 marks)**

Q5. a) A pension scheme provides a pension on ill-health retirement of 1/80th of Final Pensionable Salary for each year of pensionable service subject to a minimum pension of 20/80ths of Final Pensionable Salary. Final Pensionable Salary is defined as the average salary earned in the three years before retirement. Normal retirement age is 65 exact.
Derive a formula for the present value of the ill-health retirement benefit for a member currently aged 35 exact with exactly 10 years past service and salary for the year before the calculation date of Kshs 20,000.
(10 marks)

b) A population is subject to two modes of decrement α and β where ${q_x}^{\beta} = \frac{1}{3} + \frac{1}{4}{q_x}^{\alpha}$. Derive from first principles ${(aq)_x}^{\beta}$. State clearly any assumptions you make. **(10 marks)**

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