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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 300: ACTUARIAL MATHEMATICS I

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Differentiate between annuity-due and immediate annuity. **3marks**

b) i) By definition or otherwise, differentiate prospective reserve from retrospective reserve. **(3 marks)**

ii) State the condition under which the prospective reserve would be equal to the retrospective reserve. **(2 marks)**

c) Using the AM92 mortality table, look up A_{65} and $\ddot{a}_{65}$ at 4% p.a interest. Hence verify that A_{65} =1-d $\ddot{a}_{65}$ **3marks**

d) Calculate $\ddot{a}^{(4)}_{50:\overline{15}|}$ values for interest using AM92 mortality and 4% pa (5mks) **(5 marks)**

e) Prove that the variance of $\bar{a}_{\overline{T_x}|}$ is:

$$\text{var}\left(\bar{a}_{\overline{T_x}|}\right) = \frac{1}{\delta^2}\left[{}^2\bar{A}_x - \left(\bar{A}_x\right)^2\right]$$

(6 marks)

f) Calculate the expected present value and variance of the present value of an endowment assurance of 1 payable at the end of the year of death for a life aged 40 exact, with a term of 15 years.

Basis:

Mortality	AM92 Select
Rate of interest	4% per annum
Expenses	Nil

(8 marks)

Q2. a) Briefly explain why we hold reserves in life insurance companies **(4 marks)**

b) Evaluate the following functions, assuming the given basis:

i) $\ddot{a}_{65:\overline{20}|}$ AM92 Ultimate mortality and interest at 4% pa

ii) $A_{68:\overline{2}|}$ AM92 Ultimate mortality and interest at 6% pa **(4 marks)**

c) Let X be a random variable representing the present value of the benefits of a whole life assurance, and Y be a random variable representing the present value of the benefits of a temporary assurance with a term of n-years. Both assurances have a sum assured of 1 payable at the end of the year of death and were issued to the same life aged x.

i) Describe the benefits provided by the contract which has a present value represented by the random variable X-Y

ii) Show that

$$Cov(X,Y) = {}^{2}A^{1}_{x:\overline{n}|} - A_x * A^{1}_{x:\overline{n}|}$$

And hence or otherwise that

$$Var(X-Y) = {}^{2}A_x - ({}_{n|}A_x)^2 - {}^{2}A^{1}_{x:\overline{n}|}$$

Where the functions A are determined using an interest rate rate of I, and the functions ${}^{2}A$ are determined using an interest rate of $i^2 - 2i$

(12marks)

Q3. a) In the context of net premiums for endowment assurance, explain the following premium symbols

i) $$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

ii) $$P\left(\overline{A}_{x:\overline{n}|}\right) = \frac{\overline{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

iii) $$P^{(m)}_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}}$$

(3marks)

b) A population with limiting age 100 has the following survival function:

$$ {}_t p_0 = \left(1 - \frac{t}{100}\right)^{1/2} \qquad \text{for } 0 \le t \le 100 $$

Calculate the complete expectation of life at age 50 **(5marks)**

c) Derive a formula for the variance of the profit earned by an insurance company offering an n-year endowment assurance policy to lives aged *x*. Assume that premiums are payable annually in advance and death benefits are payable at the end of the year of death. **(8marks)**

d) A life aged exactly 33 purchases a whole life assurance policy with a sum assured of £40,00 payable at the end of the year of death. Premiums of £520 are payable annually in advance. Calculate the variance of the insurer's profit on this contract, assuming AM92 Ultimate mortality and 4% pa interest. **(4marks)**

Q4. a) Explain why premiums are normally paid in advance for an insurance policy **(2marks)**

b) Prove that:

i) For temporary annuities:

$$ \bar{a}_{x:\overline{n}|} \cong \ddot{a}_{x:\overline{n}|} - \frac{1}{2}(1 - v^n {}_n p_x) $$ **(4 marks)**

c) A level annuity of 1 pa is to be paid continuously to a 40 year-old male. On the basis of 4% pa interest and AM92 Ultimate mortality, calculate the expected present value of this annuity. **(2 marks)**

d) A life aged exactly 50 buys a 15-year endowment assurance policy with a sum assured of £50,000 payable on maturity or at the end of the year of earlier death. Level premiums are payable monthly in advance. Calculate the monthly premium assuming AM92 Ultimate mortality and 4% pa interest. Ignore expenses. **(6 marks)**

e) Prove that

$$ {}_{n|}A_x = A_x - A_{\overline{x:n}|} = v^n {}_n p_x A_{x+n} $$ **(6 marks)**

Q5. a) Prove that $\int_0^1 v^t dt = \frac{iv}{\delta}$ **(3 marks)**

Hence or otherwise, By considering a term assurance policy as a series of one-year deferred term assurance policies, show that:

$$\overline{A}^{1}_{x:n\rceil} = \frac{i}{\delta} A^{1}_{x:n\rceil}$$

(6 marks)

b) Calculate the annual premium for a term assurance with a term of 10 years to a male aged 30, with a sum assured of £500,000, assuming AM92 Ultimate mortality and interest of 4% pa. Assume that the death benefit is paid at the end of the year of death. **(7marks)**

c) John aged exactly 35, buys a term assurance policy that pays a benefit of £100,000 at the end of the year of his death if he dies before age 65. What is the expected accumulated value of this benefit at time 10?
Basis: AM92 Ultimate, 6% pa Interest **(5marks)**

END