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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 201: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define the UDD assumption and hence prove that ${}_tq_x \approx tq_x$ **6 marks**

b) Define the following terms:

i) Basis
ii) Endowment assurance
iii) Annuity-due **3 marks**

c) Define and calculate

$${}_{5/9}q_{[40]+1}$$

Basis: AM92 Select **6 marks**

d) A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value of benefits payable under this contract.

Basis: Mortality: AM92 Select
Interest: 4% per annum **6marks**

e) A graph of $f_0(t)$, the probability density function for the random future lifetime, is plotted on the vertical axis, with t plotted on the horizontal axis, for data taken from the English Life Table No. 15(Males)

You are given that $f_0(t) = {}_t p_0 \mu_t$. You observe that the graph rises to a peak at around t=80 and then falls. Explain why the graph falls at around t=80

3marks

f) Calculate $\ddot{a}_{40:4\rceil}$

Basis:

From the following life table extract

X	l_x
40	100,000
41	99,200
42	98,100
43	96,700
44	94,700

Interest 5% per annum **6 marks**

Q2. a) i) In the context of random variables, define T_X and K_x **2marks**

ii) A person is aged exactly 55 years old. Suppose that she dies when she is aged 76 years and 197 days old. What are the values of T_{55} AND K_{55} for this person? **2marks**

b) The mortality of a certain population is governed by the life table function $l_x = 100-x$, $0 \leq x < 100$. Calculate the values of the following expressions:

i) μ_{30}

ii) ${}_{10}p_{30}$

iii) $P(T_{30} < 20)$

iv) $P(K_{30} = 20)$

v) $\overset{o}{e}_{30}$ **9marks**

c) Calculate $A_{50:4\rceil}$

Basis:

Mortality $q_{50} = 0.05$

$q_{51} = 0.06$

$q_{51+t} = 1.1 q_{50+t}$ for $t \geq 1$

Interest 6% p.a. **7marks**

Q3. a) Calculate: ${}_{12}p_{[50]+1}$

Basis: AM92 Mortality **2marks**

b) If T_x and K_x are random variables measuring the complete and curtate future lifetimes, respectively, for a life aged x, write down expressions for the following symbols in terms of expected values.

i) A_x
ii) $\bar{A}^{1}_{x:\overline{n}|}$
iii) $A^{\frac{1}{}}_{x:\overline{n}|}$
iv) $\bar{a}_x$
v) $\ddot{a}_{x:\overline{n}|}$ **5 marks**

c) Calculate the values for the following functions, assuming AM92 mortality:

i) $\ddot{a}_{23:\overline{18}|}$
ii) $\frac{D_{50}}{D_{40}} a_{50}$ **8marks**

d) A whole life assurance provides a benefit of 100,000 payable immediately on the death of a male life who is now aged 45 exact.
Calculate, showing all your workings: the EPV of this policy.
Basis:
Mortality AM92 Ultimate
Rate of interest 4% p.a. **5marks**

Q4. a) Give a different example of selection shown by each of the following mortality tables:

i) ELT15
ii) PMA92C20
iii) AM92 **3marks**

b) You are given that $p_{80} = 0.988$. Estimate $_{0.5}p_{80}$ assuming:

i) A uniform distribution of death between integer ages
ii) A constant force of mortality between integer age **4marks**

c) The table below is part of a mortality table used by a life insurance company to calculate survival probabilities for a special type of life insurance policy.

d)

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}
51	1537	1517	1502	1492	1483
52	1532	1512	1497	1487	1477
53	1525	1505	1490	1480	1470
54	1517	1499	1484	1474	1462
55	1512	1492	1477	1467	1453

i) Calculate the probability that a policy holder who was accepted for insurance exactly 2 years ago and is now aged exactly 55 will die at age 57 next birthday. **4 marks**

ii) Calculate the corresponding probability for an individual of the same age who has been a policyholder for many years. **3 marks**

iii) Comment on your answers to (i) and (ii). **2marks**

e) Calculate the exact value of $\overline{A}^{1}_{70:\overline{1}|}$ assuming the force of mortality is constant between consecutive integer ages.
Basis: Mortality: ELT15 (Males)
Interest: 7.5% per annum **6 marks**

Q5. a) Explain what the following represent:

i) $l_{[x]+1}$

ii) d_x **2 marks**

b) An assurance contract provides a death benefit of £1,000 payable immediately on death.
The following basis is used:
Force of mortality: $\mu_x = 0.05$ for all x
Force of interest: $\delta = 0.04$
Calculate the EPV. **6marks**

c) A population is subject to a constant force of mortality of 0.015.
Calculate:
i) The probability that a life aged 20 exact will die before age 21.25 exact.
ii) The curtate expectation of a life aged 20 exact, **6marks**

d) Evaluate the following functions, assuming the given basis:
i) $\ddot{a}_{65:\overline{20}|}$ AM92 Ultimate mortality and interest at 4% pa
ii) $A_{68:\overline{2}|}$ AM92 Ultimate mortality and interest at 6% pa **6marks**

END