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MAIN EXAMINATION

AUGUST – DECEMBER 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 200: FINANCIAL MATHEMATICS I

Date: DECEMBER 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) It is known that the accumulation function $a(t)$ is of the form $at^2 + b$.If$100 invested at time 0 accumulates to $172 at time 3, find the accumulated value at time 10 of $100 invested at time 5. **(5marks)**

b) An investment of $10,000 is made into a fund at time $t = 0$. The fund develops the following balances over the next four years.

t	$A(t)$
0	10,000.00
1	10,600.00
2	11,130.00
3	11,575.20
4	12,153.96

i) If $ 5,000is invested at time $t = 2$ under the same interest rate environment find the accumulated value of $5,000 at time $t = 4$.

ii) Find the effective rate of interest for year 3 and year 4.

iii) Find the effective rate of discount for the fourth year. **(5marks)**

c) It is known that an investment of \$500 will increase to \$4,000 at the end of 30 years. Find the sum of the present value of three payments of \$10,000 each of which will occur at the end of 20, 40, and 60 years. **(3marks)**

d) i) Find d_5 if the rate of simple interest is 10%.

ii) Find d_5 if the rate of simple discount is 10%.

e) The present value of two payments of \$100 each to be made at the end of n years and $2n$ years is \$100. If $i = 0.08$, find n **(4marks)**

f) The cash price of an automobile is \$10,000. The buyer is willing to finance the purchase at 18% convertible monthly and to make payments of \$250 at the end of each month for four years. Find the down payment that will be necessary. **(5marks)**

g) At what annual effective rate of interest is the present value of a series of payments of \$1 every six months forever, with the first payment made immediately, equal to \$10 **(4marks)**

Q2. a) i) Show that

$$\frac{d^3}{(1-d)^2} = \frac{(i-d)^2}{1-v}$$

ii) If i and d are equivalent rates of simple interest and discount over t periods show that

$$i - d = idt$$ **(5marks)**

b) A deposit of \$1,000 is invested at simple interest at time $t = 0$. The rate of simple interest during year t is equal to 0.01t for $t = 1,2,3,4 \; and \; 5$. Find the total accumulated value of this investment at time $t = 5$. **(3marks)**

c) Adeposits$X$ into savings account at time 0 which pays interest at a nominal rate of i compounded semiannually. Bdeposits$2X$ into a different savings account at time 0 which pays simple interest at annual rate of i. A and$B$ earn the same amount of interest during the last six months of the 8th year. Calculate i. **(6marks)**

d) A and B each open up new bank accounts at time 0. Adeposits \$100 while B deposits \$50. Each account earns an effective discount rate of d. The amount of interest earned in A's account during the 11th year is equal to X. The amount of interest earned in B's account during the 17th year is also equal to X. Calculate X. **(6marks)**

Q3. a) At a certain interest rate the present value of the following two payment patterns are equal.

i) \$200 at the end of 5 years plus \$500 at the end of 10 years
ii) \$400.94 at the end of 5 years.

At the same interest rate \$100 invested now plus \$120 invested at the end of 5 years will accumulate to P at the end of 10 years. Calculate P. **(5marks)**

b) You invest \$3000 today and plan to invest another \$2000 two years from today. You plan to withdraw \$5000 in n years and another \$5000 in $n+5$ years, exactly liquidating your investment account at that time. If the effective rate of discount is equal to 6% find n. **(5marks)**

c) It is known that an investment of \$1000 will accumulate to \$1825 at the end of 10 years. If it is assumed that the investment earns simple interest at rate i during the 1st year, $2i$ during the 2nd year, …….,$10i$ during the 10th year, find i. **(4marks)**

d) You deposit \$1000 into a bank account. The bank credits interest at a nominal annual rate of i convertible semiannually for the first 7 years and a nominal annual of $2i$ convertible quarterly for all years thereafter. The accumulated amount in the account at the end of 5 years is X.The accumulated amount in the account at the end of 10.5 years is \$1980. Calculate X to the nearest dollar. **(6 marks)**

Q4. a) Find $\ddot{a}_8$ if the effective rate of discount is 10%. **(3marks)**

b) A worker aged 40 wishes to accumulate a fund for retirement by depositing \$3000 at the beginning of each year for 25 years. Starting at age 65 the worker plans to make 15 annual withdrawals at the beginning of each year. Assuming that all payments are certain to be made, find the amount of each withdrawal starting at age 65 to the nearest dollar, if the effective rate of interest is 8%during the first 25 years but only 7% thereafter. **(6marks)**

c) Consider $a_{\overline{n+k|}}$ where n is a positive integer and $0 < k < 1$. Derive a formula for $a_{\overline{n+k|}}$ where a smaller payment than the regular payments is made

i) at time $n+k$
ii) on the date of the last regular payment (c.f. Balloon payment)
iii) one period after the last regular payment (c.f. Drop payment)

d) An investment of \$1000 is to be used to make payments of \$100 at the end of every year for as long as possible. If the fund earns an annual effective rate of 5%, find how many regular payments can be made and find the amount of the smaller payment:

i) to be made on the date of the last regular payment

ii) to be made one year after the last regular payment

iii) to be made during the year following the last regular payment **(6marks)**

Q5. a) i) Show that the present value of an annuity which pays 1at the end of each k interest conversion periods, i being the rate of interest per interest conversion period and n the term of the annuity measured in interest conversion periods, is given by $\frac{a_{\overline{n}|}}{s_{\overline{k}|}}$.

ii) Deduce the formula for the present value of an annuity with similar features as in a(i) above but with payments made at the beginning of each k interest conversion periods. **(6marks)**

b) An investment of $1000 is to make payments of $100 at the end of each year for as long as possible with a smaller final payment to be made at the time of the last regular payment. If interest is 7% convertible semiannually, find the number of payments and the amount of the total final payment. **(8marks)**

c) Consider an annuity immediate with a term of n periods in which payments begin at P and increase by Q per period thereafter, i being the interest rate per period, see time diagram below:

	P	$P+Q$	$P+2Q$	$P+(n-2)Q$	$P+(n-1)$	: payment
0	1	2	3	$n-1$	n	: time

Show that the present value, A, of the annuity Is given by

$$A = Pa_n + Q\frac{a_n - nv^n}{i}$$

Hence or otherwise deduce expressions for

$i)(Ia)_{\overline{n}|}$, increasing annuity in which $P = 1$and $Q = 1$.

$ii)(Da)_{\overline{n}|}$, decreasing annuity in which $P = n$ and $Q = -1$ **(6marks)**

END