THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

MAY – JULY 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

PART TIME PROGRAMME

MAT 335: METHODS I

Date: JULY 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Show that L $\{e^{-kt}\} = \frac{1}{s+k} \quad s > k$ **(3marks)**

b) Find the Laplace Transform of a piecewise continuous function
$g(t) = \begin{cases} 2t & 0 \leq x \leq 3 \\ -1 & t > 3 \end{cases})$ **(5marks)**

c) Determine the singular points and their nature for the following differential equation
$2x^2 \frac{d^2y}{dx^2} + 7(x+1)\frac{dy}{dx} - 3y = 0$ **(5marks)**

d) Prove that $\Gamma(n+1) = n\Gamma n$ **(4marks)**

e) Using the definition of the Beta function evaluate
$\int_0^1 (1-x^3)^{-1/2} dx = \frac{1}{3}\beta\left(\frac{1}{3}, \frac{1}{2}\right)$ **(4marks)**

f) Solve the following differential equation using power series
$\frac{d^2y}{dx^2} + x^2 y = 0$ **(4marks)**

g) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ **(5marks)**

Q2. a) Prove that $\int_0^{\frac{\pi}{2}} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$ **(6marks)**

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{8}{3}}\theta \sec^{\frac{1}{2}}\theta d\theta$ **(6marks)**

c) Show that the functions$f_1(x) = 1\ and f_2(x) = x$ are orthogonal on set (-1, 1) and determine the constants A and B so that the function $f_3(x) = 1 + Ax + Bx^2$ is orthogonal to $f_1(x) and f_2(x)$ on the interval (-1, 1) **(8marks)**

Q3. a) Find the inverse Laplace Transform of $\frac{2s^2-4}{(s+1)(s-2)(s-3)}$ **(8marks)**

b) Solve the Bessel's differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$ **(12marks)**

Q4. a) (b) Solve the initial value problem using the Laplace Transform

$$y'' + 2y' + 5y = 10\ y(0) = 1\ and\ y' = 0$$ **(8marks)**

b) Find the Fourier series of the function $f(x) = x\sin x$ in the interval $(-\pi, \pi)$. Hence deduce that $\frac{\pi}{4} = \frac{1}{1.2} + \frac{1}{1.3} + \frac{1}{3.5}$ **(12marks)**

Q5. a) Using the definition of the Gamma function, evaluate

$$I = \int_0^{\infty} \sqrt[4]{X}\, e^{-\sqrt{x}} dx$$ **(6marks)**

b) The vibration of a vibrating string is governed by the equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

The length of the string isπ and the ends are fixed. The initial velocity is zero and initial deflection is $u(x, 0) = 2[\sin x + \sin 3x]$. Find the deflection $u(x, t)$ of the vibrating string for $t > 0$ **(14marks)**

END