THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

MAY – JULY 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 130: FOUNDATIONS OF APPLIED MATHEMATICS

Date: JULY 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Given the vectors 2i +j-3k and i+5j+6k. Find

i) A vector perpendicular to both of them **(3marks)**

ii) The angle between them. **(3marks)**

b) Work out 1100 +0111 **(2marks)**

c) Find the volume of the parallelepiped whose adjacent edges are represented by the vectors a=(1,2,3) b=(4,5,6) and c =(7,8,0) **(4marks)**

d) Verify $\sinh x + \cosh x = e^x$ **(4marks)**

e) Given A= $(x^2yzi - 2xz^3j + xz^2k \ and \ B = 2zi + yj - x^2k$ evaluate $\frac{\partial^3(A\times B)}{\partial y\partial x\partial z}$ at (1, 0,-2) **(4marks)**

f) Convert the following numbers to octal

(i) 1011010_2

(ii) 1110011001100_2 **(6 marks)**

g) Express $\frac{(6+i)(2-i)}{(4+3i)(1-2i)}$ $in \ the \ form \ a + ib$. **(4marks)**

Q2. a) Prove $A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$ **(5marks)**

b) Given $r_1 = 2i - j + k \quad r_2 = i + 3j - 2k \quad r_3 = -2i + j - 3k$ and $r_4 = 3i + 2j + 5k$

Find the scalars a, b and c such that $r_4 = ar_1 + br_2 + cr_3$ **(5marks)**

c) A particles moves along the curve $x = 2t^2, y = t^2 - 4t \ , z = 3t - 5$ where t is the time. Find the component of its velocity and acceleration at time $t = 1$ in the direction $i - 3j + 2k$ **(6marks)**

d) Use the definition of the hyperbolic function $\cosh\theta$ and $\sinh\theta$ to show that $\cosh^2\theta - \sinh^2\theta = 1$ **(4marks)**

Q3. a) Given the space curve $x = \sin 2t \, , y = \cos 2t \ \ z = t$. Find

i) Unit vector T and curvature K. Hence determine the unit normal vector N **(4marks)**

ii) Binormal vector B and the torsion τ **(4marks)**

b) Given that $z_1 = 1 + 3j \quad z_2 = 2 + 5j \quad z_3 = -3 - 4j$. Determine in terms of $a + bj$

i) $z_1 + z_2 - z_3$ **(4marks)**

ii) $\frac{z_1 z_2}{z_1 + z_2}$ **(4marks)**

iii) $z_1 z_2 z_3$ **(4marks)**

Q4. a) Given the space curve $x = t, y = t^2 \ z = \frac{2}{3}t^3$. Find

i) Unit tangent vector T and curvature K. Hence determine the unit normal vector N.

ii) Binormal vector B and the torsion τ **(10 marks)**

b) Convert the following to decimal

$(i) \ \ 1100_2$

$(ii) \ 16_8$

$(iii) \ \ 111100.01101_2$ **(6marks)**

c) Prove that the radius of the curvature of the curve with parametric equations

$$x = x(s), y = y(s), z = z(s) \text{ is given by}$$

$$\rho = \left[\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2\right]^{-1/2}$$ **(4marks)**

Q5. a) The path of particle moving on a curve in space has its position vector $r = 3\cos ti + 3\sin tj + t^2k$. Find

i) velocity

ii) acceleration **(4marks)**

b) Show that the acceleration α of a particle which travels along a space curve with velocity v is given by $\alpha = \frac{dv}{dt}T + \frac{v^2}{\rho}N$ **(4marks)**

c) A particle moves so that its position vector is given by $r = \cos\omega ti + \sin\omega tj$ where ω is a constant. Show that

i) the velocity v of the particle is perpendicular to r

ii) the accelerationα is directed towards the origin and has magnitude proportional to the distance from the origin

iii) $r \times v = a\ constant\ vector$. **(8marks)**

d) Express $\sqrt{3-i}$ in polar form**.** **(4marks)**

END