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MAIN EXAMINATION

MAY – JULY 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 111: DISCRETE MATHEMATICS

Date: JULY 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Let $\boldsymbol{p}$ and $\boldsymbol{q}$ be the statements:
$\boldsymbol{p}$: He is rich
$\boldsymbol{q}$: He is generous
Write the following statements in symbols forms (in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$);

i) 'He is poor but generous' **(2 marks)**
ii) 'He is rich but not generous' **(2 marks)**
iii) 'He is not generous but rich' **(2 marks)**
iv) 'He is poor and not generous' **(2 marks)**

b) Copy and complete the truth table below; **(6 marks)**

p	q	p^q	pᵛq	p → q
1	1			
1	0			
0	1			
0	0			

c) Show that $(\boldsymbol{p} \longrightarrow \boldsymbol{r}) \vee (\boldsymbol{q} \longrightarrow \boldsymbol{r})$ is logically equivalent to $(\boldsymbol{p} \wedge \boldsymbol{q}) \longrightarrow \boldsymbol{r}$, **without using a truth table**. **(8 marks)**

d) Find the bitwise **OR**, bitwise **AND**, and bitwise **XOR** of each of these pairs of bit strings.

i) 00 0111 0001, 10 0100 1000 **(4 marks)**
ii) 11 1111 1111, 00 0000 0000 **(4 marks)**

Q2. a) Explain, without using a truth table, why $(p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$ is true when at least one of **p**, **q**, and **r** is true and at least one is false, but is false when all three variables have the same truth value.

b) Let **p** and **q** be two prepositions. Where **p** and **q** represent the statements;
p: "The sun is shining".
q: "It is raining".

i) Determine the conjunction and disjunction of the propositions **p** and **q**. **(2 marks)**

ii) Draw the truth tables for both the conjunction and disjunction for the propositions. **(4 marks)**

c) i) Given two propositions **p** and **q,** define a bi-conditional statement of **p** and **q**. **(2 marks)**

ii) Show that the bi-conditional statement ($p \leftrightarrow q$), of **p** and **q** has the same truth value as; $(p \to q) \land (q \to p)$. **(4 marks)**

Q3. a) Show that the contrapositive ($\neg q \to \neg p$), of a conditional statement ($p \to q$), always has the same truth value as $p \to q$. **(8 marks)**

b) Determine whether each of these **conditional statements** is **true** or **false.** State with supporting reasons;

i) "If one plus one is equal to two, then two plus two is equal to five". **(2 marks)**
ii) "If one plus one is equal to three, then two plus two is equal to four". **(2 marks)**
iii) "If one plus one is equal to three, then two plus two is equal to five". **(2 marks)**

c) Copy and complete the table for the bit operators ***OR***, ***AND,*** and ***XOR***. **(6 marks)**

x	y	$\mathbf{x} \vee \mathbf{y}$	$x \wedge y$	$x \oplus$y
0	0			
0	1			
1	0			
1	1			

Q4. a) i) What are the **contrapositive**, the **converse** and the **inverse** of the conditional statement “The home team wins whenever it is raining”? **(2 marks)**

ii) Draw the truth tables for the contrapositive, the converse and inverse of the conditional statement in **(i)**. **(6 marks)**

b) Write each of these propositions in the form **“p if and only if q” in English.**

i) If it is hot outside you buy an ice cream cone and if you buy an ice cream cone it is hot outside. **(2 marks)**

ii) For you to win the contest it is necessary and sufficient that you have the only winning ticket. **(2 marks)**

iii) You get promoted only if you have connections and you have the connections only if you get promoted. **(2 marks)**

iv) If you watch television your mind will decay, and conversely.

iv) The trains runs late on exactly those days when I take it. **(2 marks)**

c) The De Morgan’s law states that $\sim(\boldsymbol{p} \vee \boldsymbol{q})$is logically equivalent to $\sim \boldsymbol{p} \wedge \sim \boldsymbol{q}$. Proof the law with the aid of a truth table. **(2 marks)**

Q5. a) Construct a truth table for each of the following compound propositions;

i) $(\mathbf{p} \leftrightarrow \mathbf{q}) \oplus (□\mathbf{p} \leftrightarrow \mathbf{q})$ **(4 marks)**

ii) $(\mathbf{p} \leftrightarrow \mathbf{q}) \oplus (□\mathbf{p} \leftrightarrow □\mathbf{r})$ **(4 marks)**

b) Let **p** and **q** be two propositions. Define the following terms and hence draw a truth table for each of them.

i) Conjunction of p and q. **(4 marks)**

ii) Disjunction of p and q. **(4 marks)**

iii) Implication of p and q. **(4 marks)**

END