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MAIN EXAMINATION

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 432: PARTIAL DIFFERENTIAL EQUATION I

Date: APRIL 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Classify each of the following equations as ordinary or partial differential equations; state the order and degree of each equation; and determine whether the equation under consideration is linear or non-linear. **(6marks)**

i. $x^2dx + y^2dy = xe^{-x}$

ii. $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$

iii. $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$

b) Find a PDE by eliminating a and c in $x^2 + y^2 + (z - c)^2 = a^2$ **(3marks)**

c) Show that the complete integral of $z = px + qy - 2p - 3q$ represents all possible planes through the points (2,3,0), also find the envelop of all planes represented by complete integral **(5marks)**

d) State the necessary condition for integrability of an equation the form

$$Pdx + Qdy + Rdz = 0$$ **(1mark)**

e) Solve $x_2x_3p_1 + x_3x_1p_2 + x_1x_2p_3 = x_1x_2x_3$ **(4marks)**

f) Use Jacobi's method to find a complete integral of $p_1x_1 + p_2x_2 = p_3^2$ **(6marks)**

g) Find the complete integral $x(1+y)p = y(1+x)q$ **(5marks)**

Q2. a) Find the integral surface of the $x^2p + y^2q + z^2 = 0$ which passes through the hyperbola
$xy = x + y, and\ z = 1$ **(5marks)**

b) Find the complete integral and singular integral $q^2 = z^2p^2(1-p^2)$
(7 marks)

c) Find the family orthogonal to $\emptyset(z(x+y)^2).,\ (x^2 - y^2))$ **(8marks)**

Q3. a) Solve $p\tan x + q\tan y = \tan z$ **(4marks)**

b) Show that $p^2 + q^2 = 1\ and\ (p^2+q^2)x = pz$ are compatible and hence solve
(8marks)

c) Form a PDE by eliminating arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$ **(4marks)**

d) Find the complete integral and singular integral for $z = px + qy + pq$
(4marks)

Q4. a) Solve the PDE $(y+z)p + (z+x)q = x + y$ **(6marks)**

b) Find the equation of surfaces satisfying $4yzp + q + 2y = 0\ and\ passing\ thrugh\ y^2 + z^2 = 1\ and\ x + z = 2$
(6 marks)

c) Prove for integrability hence solve for $xz^3dx - zdy + 2ydz = 0$
(8marks)

Q5. a) Solve $(e^xy + \cos x)dx + (e^x + e^yz)dy + e^ydz = 0$ **(5marks)**

b) Find the complete integral of $pq = x^my^nz^{2L}$ **(6marks)**

c) Form a partial differential equation by eliminating the arbitrary function (f) from the equation $(x+y+z) = f(x^2+y^2+z^2)$ **(5 marks)**

d) Solve $p + 3q = \tan(y - 3x)$ **(4marks)**

END