THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 335: METHODS OF APPLIED MATHEMATICS I

Date: APRIL 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define Laplace transform and give an example. **(2 marks)**

b) Show that $\ell\{e^{kt}\} = \frac{1}{s-k}; s > k$. **(5 marks)**

c) Distinguish between the following

i) Ordinary point and singular point. **(2 marks)**

ii) Orthogonal function and orthonormal function. **(2 marks)**

d) Find the first five non zero term of the Taylor series expression for the solution to the Initial value problem, $y'' = y^2 + x : y(1); y'(1) = -1$. **(5 marks)**

e) State at which point the following differential equations have singular points.

i) $x^2 y'' + xy' - 2y = 0$

ii) $(1-x^2)y'' - 2xy + 6y = 0$

iii) $(x^2+1)y'' + 6xy' - 2y = 0$ **(5 marks)**

f) Evaluate $\int_0^{\frac{\pi}{2}} (\sin x)^{\frac{8}{3}} (\sec x)^{\frac{1}{2}} dx$. **(5 marks)**

g) Explain the following terms

i) Legendre polynomial. **(1 mark)**

ii) Bessel function. **(1 mark)**

iii) Norm of a function. **(1 mark)**

iv) Gamma function. **(1 mark)**

Q2. a) Find the Laplace transform of the piecewise continuous function

$$g(t)=\begin{cases}2t; 0\le t<3\\ -1; t>3\end{cases}.$$ **(5 marks)**

b) Evaluate $\ell\left\{\dfrac{s+5}{s^2-2s-3}\right\}$. **(6 marks)**

c) Solve the initial value problem below using Laplace transform.

$y''+2y'+5y=10; y(0)=1; y'(0)=0$. **(9 marks)**

Q3. a) Use a power series to solve the differential equation $y'+xy=x^2-2x$ about x=0. Obtain a recursion formula for the coefficients and write the first few terms as a power series of the solution function. **(10 marks)**

b) Find a solution in power series form, centered at x=1 on the differential equation $xy''+y'+xy=0$. Obtain a recursion formula and write the first few terms. **(10 marks)**

Q4. a) Show that the functions $f_1(x)=1$ and $f_2(x)=x$ are orthogonal on set (-1,1). **(2 marks)**

b) Determine the constants A and B so that the function, $f_3(x)=1+Ax+Bx^2$ is orthogonal to $f_1(x)$ and $f_2(x)$ on the interval (-1,1). **(8 marks)**

c) Show that $\displaystyle\int_0^{\frac{\pi}{2}}\sin^m\theta\cos^n\theta d\theta=\frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$. **(10 marks)**

Q5. a) i) Express $\displaystyle\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of the Beta function and hence evaluate integral $\displaystyle\int_0^1 x^5(1-x^3)^{10}dx$. **(10 marks)**

ii) Also show that $\displaystyle\int_0^1 x^{n-1}(1-x^2)^{n-1}dx=\frac{1}{2}\beta\left(\frac{n}{2},n\right)$. **(3 marks)**

b) Obtain the Fourier series of

$$f(x)=\begin{cases} 0; -\pi < x < \dfrac{-\pi}{2} \\ x; -\pi < x < \dfrac{\pi}{2} \\ 0; \dfrac{\pi}{2} < x < \dfrac{\pi}{2} \end{cases} .$$ **(7 marks)**

END