



THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

MAIN EXAMINATION

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

PART TIME PROGRAMME

MAT 233: ORDINARY DIFFERENTIAL EQUATIONS I

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail: academics@cuea.edu

Date: APRIL 2018

Duration: 2 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

- Q1. a) Define a differential equation **(2 marks)**
- b) State the order and degree of the following differential equation
$$K^2 \left[\frac{d^2y}{dx^2} \right]^3 = \left[1 - \left(\frac{dy}{dx} \right)^2 \right]^2$$
 (2 marks)
- c) Check for exactness and solve the differential equation
$$y \sin x dx - (1 + y + \cos x) dy = 0$$
 (4marks)
- d) Solve the Bernoulli differential equation
$$\frac{dy}{dx} + y = xy^3$$
 (8marks)
- e) Solve the differential equation
$$y'' - 8y' + 16y = 0$$
 (8marks)
- f) Solve the equation
$$y''' + y' = 0$$
 (6marks)
- Q2. a) Using the method of separation of variables, solve the initial value problem
$$x \sin y dx + (x^2 + 1) \cos y dy = 0$$
 given that $y(1) = \frac{\pi}{2}$ **(10 marks)**

- b) Solve the linear differential equation $(x + 1) \frac{dy}{dx} + y = e^{3x}$ **(10 marks)**
- Q3. a) Solve the following non-homogeneous differential equation $\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 2\sin 4x$ using the method of undetermined coefficient. **(10 marks)**
- b) Solve the homogeneous equation $\frac{dy}{dx} = \frac{x-y}{x+y}$ **(10marks)**
- Q4. a) Determine if the following equations are homogeneous
- i). $y' = \frac{y+x}{x}$
- ii). $y' = \frac{2xye^{x/y}}{x^2+y^2 \sin \frac{x}{y}}$ **(6marks)**
- b) Solve the following differential equation **(8marks)**
- $$y' = \frac{2+ye^{xy}}{2y-xe^{xy}}$$
- c) A body is originally at $80^{\circ}C$. it cools down to $60^{\circ}C$ in 20 minutes. The surrounding temperature is $40^{\circ}C$. what will be the temperature of the body after 40 minutes from the origin? **(6marks)**
- Q5. a) Find the orthogonal trajectory of the family of line of slope c and passing through the point $(1,5)$ **(6marks)**
- b) The population of organisms governed by the law of simple population growth has a growth rate of 0.35 per member per week, that is $\frac{dp}{dt} = 0.35p$. How long does it take for the population to triple? **(8marks)**
- c) Determine c_1 and c_2 so that $c_1 \sin 2x + c_2 \cos 2x + 1$ will satisfy the conditions $y\left(\frac{\pi}{8}\right) = 0$ and $y'\left(\frac{\pi}{8}\right) = \sqrt{2}$ **(6marks)**

END