THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

MAIN EXAMINATION

P.O. Box 62157 00200 Nairobi - KENYA Telephone: 891601-6 Fax: 254-20-891084 E-mail:academics@cuea.edu

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 202: GROUP THEORY I

Date: APRIL 2018	Duration: 2 Hours
INSTRUCTIONS: Answer	Question ONE and any other TWO Questions

Q1.	a)	Consider C as a group under the multiplication. Where \mathbb{C} is a set of complex numbers defined by; $\mathbb{C} = \{1, i, -1, -i\}$.i)draw a Cayley table for the group of complex numbers under the group of complex numbers under the Cayley table for the group of complex numbers is an Abelian group.(4 mark numbers is an Abelian group.	s)
	b)	Define the following and give three examples of each; i) Finite group (4 mark ii) Infinite group (4 mark	•
	c)	i) Define a Cayley table and hence draw a Cayley table for a gro integer mod 4 under addition. (10 mar	-
		ii) Show whether or not the Cayley table in (<i>i</i>) is an Abelian group (2 mark	•
	d)	Consider M as a group under the multiplication. Where M is a set of matrices defined by; $M = \left\{ e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, a = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, c = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}.$:

Cuea/ACD/EXM/JANUARY – APRIL 2018 / MATHEMATICS AND COMPUTER SCIENCE

Page 1

ISO 9001:2008 Certified by the Kenya Bureau of Standards

- i) Draw a Cayley table for the (M,*).
- ii) Show that the group (M,*) is an Abelian group. (1 marks)
- Q2. a) Define a homomorphism from a groupoid (A, α) into a groupoid (B, β) . Where A and B are groupoids of positive real numbers. (4 marks)
 - b) Let *A* be the semigroup of integers under the usual addition of integers and let *B* be the semigroup of even integers under the usual addition. Verify that the mapping $\theta: A \to B$ defined by $\theta: a \to 2a$ for all $a \in A$ is a homomorphis of *A* into *B*. (8 marks)
 - c) Let *G* be the semigroup of integers under the usual addition of integers and let *H* be the semigroup of even integers under the usual addition. Verify that the mapping $\Omega : G \to H$ defined by $\Omega : g \to 2g$ for all $g \in G$ is a homomorphis of *G* into *H*. (8 marks)
- Q3. a) A group of integers under addition is defined by $(\mathbb{Z}, +)$, where \mathbb{Z} is a set of positive integers. Show that $(\mathbb{Z}, +)$ is a group. (10 marks)
 - b) A structure is defined by $(\mathbb{Q}, *)$, where \mathbb{Q} is a set of rational numbers. Show that the structure is **NOT** a group. (10 marks)
- Q4. a) Explain a subgroup.
 - b) Consider a group of positive integers under addition, $(\mathbb{Z}, +)$ and a group of even integers under addition, $(2\mathbb{Z}, +)$ to be a subgroup. Show that the subgroup $(2\mathbb{Z}, +)$ is also a group. (12 marks)
- Q5. a) Define the following terms;
 - i) Epimorphism
 - ii) Monomorphism
 - iii) Isomorphism
 - b) A Cayley table for integer mod five group, $(\mathbb{Z}_5, +)$ is shown below. Where \mathbb{Z} is a set of positive integers.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2

Page 2

ISO 9001:2008 Certified by the Kenya Bureau of Standards

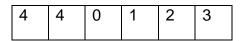
(8 marks)

(2 marks)

(2 marks)

(2 marks)

(4 marks)



Use the Carley table to show that;

- i) $(\mathbb{Z}_5, +)$ is a group.
- ii) $(\mathbb{Z}_5, +)$ is an Abelian group. ***END***

(12 marks) (2 marks)

ISO 9001:2008 Certified by the Kenya Bureau of Standards