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MAIN EXAMINATION

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 202: GROUP THEORY I

Date: APRIL 2018	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Consider C as a group under the multiplication. Where $\mathbb{C}$is a set of complex numbers defined by; $\mathbb{C} = \{\mathbf{1},\ \boldsymbol{i},\ -\mathbf{1},\ -\boldsymbol{i}\}$.

i) draw a Cayley table for the group of complex numbers under multiplication $(\mathbb{C},\ *)$. **(4 marks)**

ii) Show that the Cayley table for the group of complex numbers is an Abelian group. **(1 marks)**

b) Define the following and give three examples of each;

i) Finite group (**4 marks)**

ii) Infinite group **(4 marks)**

c) i) Define a Cayley table and hence draw a Cayley table for a group of integer mod 4 under addition. **(10 marks)**

ii) Show whether or not the Cayley table in (i) is an Abelian group. **(2 marks)**

d) Consider M as a group under the multiplication. Where M is a set of matrices defined by;$\boldsymbol{M} = \left\{\boldsymbol{e} = \begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{bmatrix}, \boldsymbol{a} = \begin{bmatrix}-\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{bmatrix}, \boldsymbol{b} = \begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & -\mathbf{1}\end{bmatrix}, \boldsymbol{c} = \begin{bmatrix}-\mathbf{1} & \mathbf{0}\\ \mathbf{0} & -\mathbf{1}\end{bmatrix}\right\}$.

i) Draw a Cayley table for the $(\boldsymbol{M},*)$. **(4 marks)**
ii) Show that the group $(\boldsymbol{M},*)$ is an Abelian group. **(1 marks)**

Q2. a) Define a homomorphism from a groupoid (A, α) into a groupoid $(B,\ \beta)$. Where A and B are groupoids of positive real numbers. **(4 marks)**

b) Let A be the semigroup of integers under the usual addition of integers and let B be the semigroup of even integers under the usual addition. Verify that the mapping $\theta: A \rightarrow B$ defined by $\theta : a \rightarrow 2a\ for\ all\ a \in A$ is a homomorphis of A into B. **(8 marks)**

c) Let G be the semigroup of integers under the usual addition of integers and let H be the semigroup of even integers under the usual addition. Verify that the mapping $\Omega : G \rightarrow H$ defined by $\Omega : g \rightarrow 2g\ for\ all\ g \in G$ is a homomorphis of G into H. **(8 marks)**

Q3. a) A group of integers under addition is defined by $(\mathbb{Z}, +)$**,** where $\mathbb{Z}$is a set of positive integers. Show that $(\mathbb{Z}, +)$ is a group. **(10 marks)**

b) A structure is defined by $(\mathbb{Q},\ *)$**,** where $\mathbb{Q}$is a set of rational numbers. Show that the structure is **NOT** a group. **(10 marks)**

Q4. a) Explain a subgroup. **(8 marks)**

b) Consider a group of positive integers under addition, $(\mathbb{Z}, +)$ and a group of even integers under addition, $(2\mathbb{Z}, +)$ to be a subgroup. Show that the subgroup $(2\mathbb{Z}, +)$ is also a group. **(12 marks)**

Q5. a) Define the following terms;
i) Epimorphism **(2 marks)**
ii) Monomorphism **(2 marks)**
iii) Isomorphism **(2 marks)**

b) A Cayley table for integer mod five group, $(\mathbb{Z}_5, +\ )$ is shown below. Where $\mathbb{Z}$ is a set of positive integers.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2

4	4	0	1	2	3

Use the Carley table to show that;

i) $(\mathbb{Z}_5, +)$ is a group. **(12 marks)**

ii) $(\mathbb{Z}_5, +)$ is an Abelian group. **(2 marks)**

END