



THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

MAIN EXAMINATION

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail: academics@cuea.edu

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 161: PROBABILITY AND STATISTICS II

Date: APRIL 2018

Duration: 2 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

- Q1. a) Define the following terms as used in statistics giving an example in each case
- i) Population **(1mark)**
 - ii) Sample **(1mark)**
 - iii) Continuous variable **(2marks)**
- b) The result of a random experiment is a binomial random variable with mean 1.2 and variance 1.08. Determine $\Pr.(X = 3)$ **(4 marks)**
- c) Write in terms of the moment generating function and its derivatives, an expression for the mean and variance of a random variable. **(3marks)**
- d) The experimental values relating Centripetal force and radius, for a mass travelling at constant velocity in a circle are as shown:

Force (N)	5	10	15	20	25	30	35	40
Radius(Cm)	55	30	16	12	11	9	7	5

Determine the equations of

- i) The regression line of force on radius **(4 marks)**
- ii) The force at a radius of 40 cm **(2 marks)**

- e) The number of mistakes counted in 100 typed pages of a typist unveiled that she made 2.8 mistakes on average per page. Determine the probability that in a page typed by her;
- There is no mistake **(2marks)**
 - There are two or less mistakes **(2marks)**
- f) Suppose $X \sim N(\mu=10, \sigma^2=4)$, compute $\Pr.(11 < X < 13)$. **(4marks)**
- g) The lifetime of a certain kind of energy cell is a random variable having a normal distribution with standard deviation $\sigma=15$ hours. Determine the mean lifetime of this cell if the probability that it will last more than 300 hours is 0.15. **(4marks)**

- Q2. a) X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

- Determine the cumulative distribution function of x and plot its graph **(4 marks)**
 - Calculate the median of the distribution **(3 marks)**
- b) In a competitive examination of 5,000 students, the marks of the examinees in statistics were found to be distributed normally with mean 45 and standard deviation 14. Determine the number of examinees whose marks, out of 100 were
- Less than 30 **(3marks)**
 - Between 60 and 80 **(2marks)**
 - More than 40 **(2marks)**

- c) A continuous random variable X has probability density function

$$f(x) = \begin{cases} a + bx, & 0 \leq x \leq 1 \\ 0, & elsewhere \end{cases}$$

If $E(X) = 1$, determine the constants a and b in $f(x)$ **(6marks)**

- Q3. a) If X is a random variable whose p.d.f is given by

$$f(x) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} < x < \sqrt{3} \\ 0, & elsewhere \end{cases}$$

Using the Chebychev's inequality, find the upper bound of

$$\Pr. [|X - \mu_x| \geq 1.5\sigma_x]$$

Where μ_x and σ_x is the mean and standard deviation of X respectively. What is the exact probability? **(6marks)**

b) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Compute $Var.(6X + 11)$ **(7marks)**

c) The relationship between monthly car sales and income from the sale of petrol for a garage is as shown:

Cars sold	2	5	3	12	14	7	3	28	14	7	3	13
Income from petrol sales (£'000)	12	9	13	21	17	22	31	47	17	10	9	11

Determine the linear coefficient of correlation between these quantities and interpret your result. **(7marks)**

Q4. a) A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample S space is given by $S = \{(b,b), (b,g), (g,b), (g,g)\}$ and all outcomes are equally likely.

Hint: $[(b,g)$ means for instance, that the older child is a boy and the younger child is a girl] **(4 marks)**

b) If the number of accidents on a highway between 9.00 am and 10.00 am on a given morning is a random variable having Poisson distribution with rate parameter $\lambda = 7.3$. Find the probability that there will be at least 34 minutes between accidents. **(5marks)**

c) In an examination, the average mark was 76.5 and the standard deviation was 9.5. If 15% of the class scored grade A and the marks are assumed to follow a normal distribution, what is the lowest possible grade A mark? **(5 marks)**

d) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{1}{18}(x+3), & -3 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Compute the mean and variance of X . **(6marks)**

Q5. a) Telkom, a telephone answering service for business executives, has found that the average telephone message is 150 seconds, with standard deviation of 15 seconds. It has also observed that the length of the messages is a normally distributed random variable. Determine.

i) The probability that one call would take between 150 and 155 seconds. **(5marks)**

ii) The probability that a random sample of 35 calls will have a mean of greater than 145 seconds. **(3marks)**

b) Let X_1 and X_2 be independent random variables each having the probability distribution

$$f(x) = \begin{cases} e^{-x}; & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the joint probability distribution function of X_1 and X_2 **(4marks)**

c) In an experiment to determine the relationship between frequency and the inductive reactance of an electrical circuit, the following results were obtained

Frequency (Hz)	Inductive reactance (Ohms)
50	30
100	65
150	90
200	130
250	150
300	190
350	200

Determine the equation of the regression line of inductive reactance on frequency, assuming a linear relationship. **(8 marks)**

END