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MAIN EXAMINATION

JANUARY – APRIL 2018 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

ACS 202: LINEAR PROGRAMMING

Date: APRIL 2018	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1. a) Define linear programming, and state the general formulation of the mathematical model, in canonical form of a linear programming problem. **(4 marks)**

b) The Whitt window company is a company with only 3 employees which makes two different kinds of hand-crafted windows: a wood framed and an aluminum-framed window. They earn $60 profit for each wood-framed window and $30 profit for each aluminum-framed window. Doug makes the wood-frames, and can make 6 per day. Linda makes the aluminum frames, and can make 4 per day. Bob forms and cuts the glass, and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass, and each aluminum-framed window uses 8 square feet of glass. The company wishes to determine how many windows of each type to produce per day to maximize total profit.

i) Formulate a linear programming model for this problem. **(3 marks)**

ii) Use the graphical procedure to solve this problem. **(5 marks)**

iii) How would the optimal solution change (if at all) if the profit per wood framed window decreases from $60 to $40? From $60 to $20? **(2 marks)**

iv) How would the optimal solution change if Doug makes only 5 wood-frames per day? **(2 marks)**

c) Define a convex set and a convex function. **(4 marks)**

d) Define a convex combination and consider the following linear programming problem;

$\text{Max } Z = 3X_1+2X_2$
Subject to the constraints
$X_1 \leq 4$
$2X_2 \leq 12$
$3X_1+ 2X_2 \leq 18$
And $X_1 \geq 0$, $X_2 \geq 0$.

Solve the Linear programming problem using the simplex method.Is there more than one optimal basic feasible solution to this problem? If there is, state the optimal basic feasible solutions obtained from the simplex method and write all other optimal solutions as a convex combination of these optimal basic feasible solutions. **(10 marks)**

Q2. a) Put the following problem in the standard form:
Maximize $Z = 2X_1 - X_2 + X_3$
Subject to the constraints
$X_1 + 3X_2 - X_3 \leq 20$
$2X_1 - X_2 + X_3 \leq 12$
$X_1 - 4X_2 - 4X_3 \geq 2$
And
$X_1 \geq 0$, X_2 and X_3 unrestricted. **(8 marks)**

b) i) When does the phenomenon of degeneracy occur in the solution of linear programming problems? **(2 marks)**

ii) Consider the following linear programming problem:
Maximize $Z = 5X_1 - 2X_2 + 3X_3$
Subject to the constraints
$2X_1 + 2X_2 - X_3 \geq 2$
$3X_1 - 4X_2 \leq 3$
$X_2 - 3X_3 \leq 5$

And
$X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$.

Clearly state the initial basic feasible solution to this problem and the leaving basic variable in the initial simplex tableau. **(10 marks)**

Q3. Use the Big M method to solve the following linear programming problem:
Minimize $Z = 0.4X_1 + 0.5X_2$

Subject to the constraints
$0.3X_1 + 0.1X_2 \leq 2.7$
$0.5X_1 + 0.5X_2 = 6$
$0.6X_1 + 0.4X_2 \geq 6$
And
$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0.$ **(20 marks)**

Q4. a) Define the term basis. **(2 marks)**

b) Find the dual of the following linear programming problem:
Maximize $Z = 3X_1 – X_2 + X_3$
Subject to the constraints
$4X_1 - X_2 \leq 8$
$8X_1 + X_2 + 3X_3 \geq 12$
$5X_1 - 6X_3 \leq 13$
And
$X_1, X_2, X_3 \geq 0.$ **(8 marks)**

c) Use the dual simplex method to solve the following linear programming problem:
Maximize $Z = -3X_1 – X_2$
Subject to the constraints
$X_1 + X_2 \geq 1$
$2X_1 + 3X_2 \geq 2$
And
$X_1X_2 \geq 0.$ **(10 marks)**

Q5. Consider the following linear programming problem:
Maximize $Z = 3X_1 + 2X_2$
Subject to the constraints
$X_1 + X_2 \geq 1$
$X_1 + X_2 \leq 7$
$X_1 + 2X_2 \leq 10$
$X_2 \leq 3$
And
$X_1\ X_2 \geq 0.$
Write the dual of the above primal linear programming problem and use the big M method to solve the dual linear programming problem.
Give the solutions to the decision variables X_1 and X_2 in the primal linear programming problem. **(20 marks)**

END