## Date: July 2016

Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and ANY OTHER TWO Questions
Q1. a) Define the following terminologies related to the algebra of set theory providing an example or illustration as appropriate.
i A set.
(1 mark)
ii Disjoint sets.
(1 mark)
iii Compliment of a set.
ii Foe each of the following (i) and (ii) draw a veun diagram using the given information to fill in the number of elements for each region (distinct sets)

I $\quad c(u)=38, c(A)=16, c(A n B)=12, c\left(B^{\prime}\right)=20$
(4 marks)
li $\quad c(A u B)=17, c(A n B)=3, C(A)=8, C\left(A^{\prime} U B^{\prime}\right)=21$
(4 marks)
NB eg $c(U)=38$ is the universal set
$\mathrm{A}^{\prime}=$ compliment of $\operatorname{set} \mathrm{A}$
iii In a survey of 10,281 people restricted to those who were either Kenyan or Male or over 18 years , a Kenyan military officer conducting a veting recruitment exercise provided the following data:

Kenyan - 3490, Kenyan males - 1745, over 18 and Kenyan - 1341

Male - 5822, over 18 and Male - 859
Kenyan male over 18-239, over 18-4722
Required:
Using set theory, determine whether the data provided by the military officer is consistent or not. Provide a clear justification for your conclusion.
(8 marks)
b) The percentage of people who have a Bcom degree in Kenya has been increasing at a constant rate and is summarized in the following table:

| Year | 1995 | 2000 | 2005 | 2010 | 2015 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Percentage with Bcom degree | 30 | 34 | 38 | 42 | 46 |

i Determine the function that relates year to the percentage of people with a Bcom degree.
ii If the trend continues, estimate the percentage of people who will have a Bcom degree by the year 2025.
iii Determine the year in which Bcom degree holders will reach $58 \%$.
c) Define the concepts of "Differential Calculus" and provide one use of the concept in a business setting.
d) Find the slope of $f(x)=x^{3}-2 x^{2}+4 x+1$ at (3, 22).
(2 marks)

Q2. a) The selling price of a commodity per unit is Khs 200 while the variable cost per unit is $70 \%$ of the selling price. The fixed cost is Kshs 800,000
i Determine the break even sales.
(3 marks)
ii Sales units required to make a profit of Kshs 2 million.
(2 marks)
b) A company wholesales a certain brand of shampoo in a particular city. Their marketing research department established the following weekly supply and demand equations.
$P=\frac{x}{450}+1 / 2$, supply equation
$P=\frac{6,300}{x}$, demand equation
Required:
Determine the equilibrium quantity and price for the market of this brand of shampoo.
(7 marks)
c) A major bank offers a credit card which can be used domestically. Data given over time indicate that the collection percentage for credit issued in any month is an exponential function of the time since the credit was issued. Specifically, the function approximating the relationship is
$p=0.9\left(1-\lambda^{-0.08 t}\right)$ where $p$ equals to the percentage of amounts receivable (in shillings collected t months after the credit is granted.
i What percentage is expected after 4 months?
(2 marks)
ii What value does p approach as tincreases without bound (limit) and hence what is the expected percentage of bad debts?
(2 marks)
d) For a certain commodity, the demand equation is given by $D=-3 P+20$. At a price of $£ 1$, four units of the commodity are supplied. If the supply equation is linear and the market equilibrium price is $£ 4$, find the supply equation.
(4 marks)
Q3. a) The market research department of a company recommends that the company manufactures and markets a new toilet air freshener for use in homes. After suitable test marketing, the research department presents the following price-demand function.
$X=10000-1000 p$ where $x$ is the number of tins (units) of the freshener of a given standard size that retailers are likely to buy at $\mathrm{K} £ p$ per unit. The financial department provides the following cost function.
$C=7000+2 x$ where $£ 7000$ is the estimate of the fixed cost (tooling and overhead) and K£2 is the estimate of variable cost per unit (diaterial, labor, transportation, etc).

Required:
i Find the revenue function of the company.
(3 marks)
ii Find the profit function of the company.
iii Find the level of production that maximizes profit and the maximum profit in $\mathrm{K} £$.
(4 marks)
iv What price should the company charge in order to maximize profit?
(2 marks)
b) Suppose the cost of production increases by $£ 0.5$ per unit because of a tax levy on the business by the government,
i) Determine the adjusted cost function.
ii) Determine the adjusted profit function and the level of production that maximizes profit, and the maximum profit in $\mathrm{K} £$
(7 marks)
Q4. a) The supply function of a commodity is quadratic and passes through the points $(30,500),(40,3600)$ and $(50,6300)$

Required:
i Determine the supply function $s=f(x)$
(10 marks)
ii Predict supply when price $(x)$ is 45
(2 marks)
b) Determine the turning point (s) of the function $y=2 x^{3}+3 x^{2}-12 x+24$, and the nature of the turning point ( $s$ ) using the second derivative. ( 8 marks)
*END*

