THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

JANUARY – APRIL 2015 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES (PHYSICS)

SCHOOL FOCUSED PROGRAM

PHY 401: QUANTUM MECHANICS II

Date: April 2015	**Duration: 2 Hours**
Instructions: Answer Question ONE and any other TWO Questions	

Q1 a) Calculate the wavelength of an electron accelerated through a potential of 100V.

(3 marks)

b) State why the $\psi = e^{-x^2}$ is not acceptable as a wavefunction.

(2 marks)

c) The functions given below are defined in the interval x = -a and x = +a as follows;

i) $F_1(x) = N_1(a^2 - x^2)$
ii) $F_2(x) = N_2(a^2 - x^2)$

Assuming the value of the function to be zero for x < -a and x > +a.

i) Calculate the normalization constant N_1.

(4 marks)

ii) Show that the functions $F_1(x)$ and $F_2(x)$ above are orthogmal.

(4 marks)

d) Show that the De Broglie wavelength may be written as:

$$\lambda = \frac{h}{\sqrt{2m(E-V)}}$$

(3 marks)

e) Show that the uncertainty in the momentum can be expressed as $\Delta p = \sqrt{<P^2> - <p>^2}$

f) Determine the L+ and L- operating on the eigenstate $|2,-1\rangle$ **(6 marks)**

g) Generate the first three Hermite polynomials given that;

$$h_\nu(z) = (-1)^\nu e^{z^2} \frac{d^\nu}{dz^\nu} e^{-z^2}$$ **(3 marks)**

Q2. a) What is the expression for the operator

$$\left(x\frac{xd}{dx}\right)^2$$ **(4 marks)**

b) Show that the function Sin3x is an eigenfunction of $\frac{d^2}{dx^2}$ hence state the eigenvalue. **(4 marks)**

c) In a region of space, a particle with mass m and zero energy has a time independent wave function.

$$\psi(x) = Axe^{-x^2/L^2}$$

Where A and L are constants. Determine the potential energy U(x) of the particle.

(7 marks)

d) Consider a free particle of mass m confined between two walls separated by a distance a. The motion of the particle along the x axis may be represented by a stationary wave whose equation is;

$\psi(x,t) = \psi_{\max}\sin\left(\frac{\pi n x}{a}\right)\sin(\omega t+\phi)$ show that the amplitude, linear momentum and hence energy of the particle is quantized.

(5 marks)

Q3. a) Show that:

i) $\left[x^n, P_x\right] = \frac{1}{2\pi i} n x^{n-1}$ **(4 marks)**

ii) $\left[x, \frac{d}{dx}\right] = 1$ **(4 marks)**

b) A certain system is decribed by the Hamiltonian operation.

$$\hat{H} = \frac{-d^2}{dx^2} + x^2$$

i) Show that $\psi = Ax\exp\left(-x^2/2\right)$ is an eigenfunction of $\hat{H}$ hence determine the elgenvalue. **(4 marks)**

ii) Determine A so that the function is normalized.

(3 marks)

iii) Determine the expectation value of x for the state described by
$\psi = Ax\exp\left(x^2/2\right)$ **(5 marks)**

Q4. A quantum particle of mass m moves in ID incident from the left on a potential step at x = 0 of height V_0 described by:

$$V_{(x)} = \begin{cases} 0 \;\; x < 0 \\ V_0 x > 0 \end{cases}$$

Consider continuum states with energy E higher than the step.

a) Set up the wave function $\psi(x)$ and apply correct boundary conditions at x = 0 **(10 marks)**

b) Determine the amplitude of the reflected and transmitted waves relative to the incidents waves. **(6 marks)**

c) Show that the incident and the transmitted probability currents and therefore the coefficient of hausmission T(E) can be expressed as a function of energy as;

$$T(E) = \frac{4\sqrt{E}\sqrt{E-V_0}}{\left(\sqrt{E}+\sqrt{E-V_o}\right)^2}$$

Q5. a) Derive expressions that connect the particle like properties; momentum, energy and kinetic energy to their corresponding wavelike properties in quantum mechanics. **(6 marks)**

b) Find the operators for the linear momentum and kenetic energy of a system. **(6 marks)**

c) The annilulation and creation operators are defined as follows:

$$\hat{a} = \left(\beta/\sqrt{2}\right)\left(\hat{x} + i\hat{p}/mw_0\right)$$

$$\hat{a}^{+} = \left(\beta/\sqrt{2}\right)\left(\hat{x} + i\hat{p}/mw_0\right)$$

$$where;\;\; \beta = \sqrt{\frac{mw_0}{\hbar}}$$

Using the fundamental conumitaln

$$\left[\hat{x}, \hat{p}\right] = i\hbar$$

i) Show that $\left[\hat{a},\hat{a}^{+}\right]=1$ **(3 marks)**

ii) Show that

$\hat{H}=\hbar w_0\left(a^{+}a+\frac{1}{2}\right)$ where $\hat{H}$ is the Hamultonian operator of a simple harmonic Oscillator. **(5 marks)**

END