THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157
00200 Nairobi - KENYA
Telephone: 891601-6
Fax: 254-20-891084
E-mail:academics@cuea.edu

MAIN EXAMINATION

JANUARY - APRIL 2015 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES (PHYSICS)

PHY 401: QUANTUM MECHANICS II

Date: April 2015	**Duration: 2 Hours**
Instructions: Answer Question ONE and any other TWO Questions.	

Q1. a) Define the following terms

i) Eigenstate **(1 mark)**

ii) Stationary state **(1 mark)**

iii) Wave function **(1 mark)**

b) Show that;

$$\left[\frac{\partial}{\partial x}, x^n\right] = nx^{n-1}$$

(3 marks)

c) Show that the commutation of the spin S_x and S_y is $i\hbar^2 S_z$ given that;

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};\ S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix};\ S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(3 marks)

d) If $|Q\rangle$ is a unit normalized state vector and U is a unitary operator, determine.

$$\langle Q|U^{+}U|Q\rangle$$ **(2 marks)**

e) Show that the De Broglie wavelength can be given as;

$$\lambda = \frac{h}{\left[2m(E-V)\right]^{1/2}}$$ **(3 marks)**

f) Consider a free particle of mass m confined between two walls separated by a distance a. The motion of the particle along the x axis may be represented by a stationary wave whose equation is;

$$\psi(x,t)=\psi_{\max} Sin\left(\frac{n\pi}{a}x\right)\sin(\omega t+\phi)$$

Show that the amplitude, linear momentum and hence energy of the particle is quantized.

(5 marks)

g) Determine the L_+ and L_- operating on the elgenstate $|2,-1>$

(4 marks)

h) Generate the first three Hermite polynomials given that:

$$h_\nu(z)=(-1)^\nu e^{z^2}\frac{d^\nu}{dz^\nu}e^{-z^2}$$

(4 marks)

Q2. a) In a region of space, a particle with mass, m and zero energy has a time - independent wave function;

$\psi(x)=Axe^{-x^2/L^2}$ Where A and L are constants. Determine the potential energy U(x) of the particle.

(7 marks)

b) Show that the uncertainty in the momentum can be expressed as;

$$\Delta P=\left[<p^2>-<p>^2\right]^{1/2}$$

(5 marks)

c) Some Na^+ ions (23 amu) are accelerated to mean kinetic energy of 200keV and travel in wave packets with $\Delta x \approx 1.0cm$;

i) Give numerical estimate for momentum dispersion.

(3 marks)

ii) Show that the energy dispersion can be given as;

$\frac{2\Delta p}{p}$ and hence give its numerical estimate.

(5 marks)

Q3. The particle on a ring experiences the following potential;

$V(x)=\varepsilon Sin^2\phi$

Determine the first order energy correction to the degenerate $m_l=\pm 1$ state and the value of the coefficients for the zero order wave function $\phi_n^{(0)}$.

(20 marks)

Q4. a) i) Show that for a particle moving in a rectangular 3D box,

$$E=\frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n^2y}{L^2y}+\frac{n_z^2}{L_z^2}\right)$$

Where n_x, n_y and n_z are the dimensions of the box.

(8 marks)

ii) If the box in 3b(i) is a cubical, determine

I. E_{111} **(3 marks)**

II. ψ_{111} **(3 marks)**

b) A particle of mass m = 1.0 x 10^{-26}g is confined to move in a box of length 2.0 $\overset{0}{A}$. Calculate for n = 1 and n = 2 the probability of finding the particle between 1.6000 $\overset{0}{A}$ and 1.6001 $\overset{0}{A}$.

(7 marks)

Q5. a) A quantum particle of mass m moves in ID incident from the left on a potential step at x = 0 of height V_0 described by:

$$V_{(x)} = \begin{cases} 0, & x < 0 \\ V_0 & x > 0 \end{cases}$$

Consider continuum states with energy E higher than the step E>V_0

i) Determine the amplitude of the reflected and transmitted waves relative to the incident waves.

(10 marks)

ii) Show that the incident and transmitted probability currents and therefore the coefficient of transmission T(E) can be expressed as a function of energy as:

$$T(E) = \frac{4\sqrt{E}\sqrt{E - V_0}}{(\sqrt{E} + \sqrt{E - V_o})^2}$$ **(3 marks)**

c) Consider the state vector;

$|\psi(t = 0) = A(|2,1\rangle + 3|1,-1\rangle)$

i) Normalize the state vector. **(43marks)**

ii) What are the possibilities and probabilities of a measurement of L^2?

(4 marks)

END