

THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

MAIN EXAMINATION

JANUARY-APRIL 2024

FACULTY OF SCIENCE

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DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 463: OPERATIONS RESEARCH III

DATE: APRIL 2024

Duration: 2 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

Q1.

- a) What is an economic model. (3 Marks)
- b) Define goal deviation and name the two types of goal deviation. (4 Marks)
- c) Let $X_0 \sim (\frac{3}{4}, 0, \frac{2}{4}, 0, 0, 0, 0)$. What is the probability of the trajectory 1,2,3,2,3,4? (3 Marks)
- d) Alpha company produces two kinds of fancy products, pen holder and paper tray. Production of either of them requires 1 hr production capacity in the plant. The plant has a maximum production capacity of 12 hrs per week. The maximum number of pen holders and paper trays that can be sold are 7 and 10 respectively. The gross margin from the sales of pen holder is Rs 90 and Rs 45 for a paper tray. The overtime hours should not exceed 3 hrs per week if required. The plant manager has set the following goals in order of importance:
- P1: He wants to avoid any under-utilization of production capacity
 - P2: He wants to limit the overtime hours to 3 hrs
 - P3: He wants to sell as many pen holders and paper trays as possible. Since the gross margin from the sale of a pen holder is set at twice the amount of the profit from a paper tray, the manager has twice as much desire to achieve the sales goal for pen holders as for paper trays.
 - P4: The manager wishes to minimize the overtime operation of the plant as much as possible.
- (10 Marks)
- e) A textile company produces two types of materials A and B. Material A is produced according to direct orders from furniture manufacturers. The material B is distributed to

retail fabric stores. The average production rates for material A and B are identical at 1000 meters/hour. By running two shifts the operational capacity of the plant is 80 hours per week. The marketing department reports that the maximum estimated sales for the following week is 70000 meters of material A and 45000 meters of material B. According to the accounting department the profit from a meter of material A is Rs. 2.50 and from a meter of material B is Rs. 1.50. The management of the company decides that a stable employment level is the primary goal for the firm. Therefore, whenever there is demand exceeding normal production capacity, management simply expands production capacity by providing overtime. However, management feels that overtime operation of the plant of more than 10 hours per week should be avoided because of the accelerating costs. The management has the following goals in the order of importance:

- The first goal is to avoid any under-utilization of production capacity.
- The second goal is to limit the overtime operation of the plant to 10 hours.
- The third goal is to achieve the sales goals of 70000 and 45000 respectively for both the materials.
- The last goal is to minimize the overtime operation of the plant as much as possible.

Formulate this as a goal programming problem to help the management for the best decision and solve the problem using simplex method. **(10 Marks)**

Q2.

a) Name the two types of goal programming formulations. **(2 Marks)**

b) The manufacturer of a certain product is considering the purchase of one of three different packaging systems. The product sells for \$10, and the production cost (excluding packaging cost) is \$5 per unit. The cost data for the three packaging systems are

System No.:	Purchase cost	Variable cost per unit of product	Scrap value
1	\$100	\$1.50	\$10
2	200	1.00	20
3	400	0.50	40

All three systems last one year only and will then be sold at the listed salvage value. The demand for the product over the year can be regarded as a random variable with the following probability distribution:

Demand, x	Probability, $p(x)$
100	0.3
200	0.6
400	<u>0.1</u>
	1.0

Which system should be bought?

(11 Marks)

- c) A product is manufactured by a certain automatic machine in batches of 3 items. The selling price is \$10, and the production cost is \$5 per item. Before each batch is produced, the machine may be adjusted by a skilled mechanic at a cost of \$6 per batch. If the machine is so adjusted, there will be no defectives in the batch. If it is not adjusted, some items may be defective; on the basis of past experience, the probability distribution of the number of defectives is estimated as follows

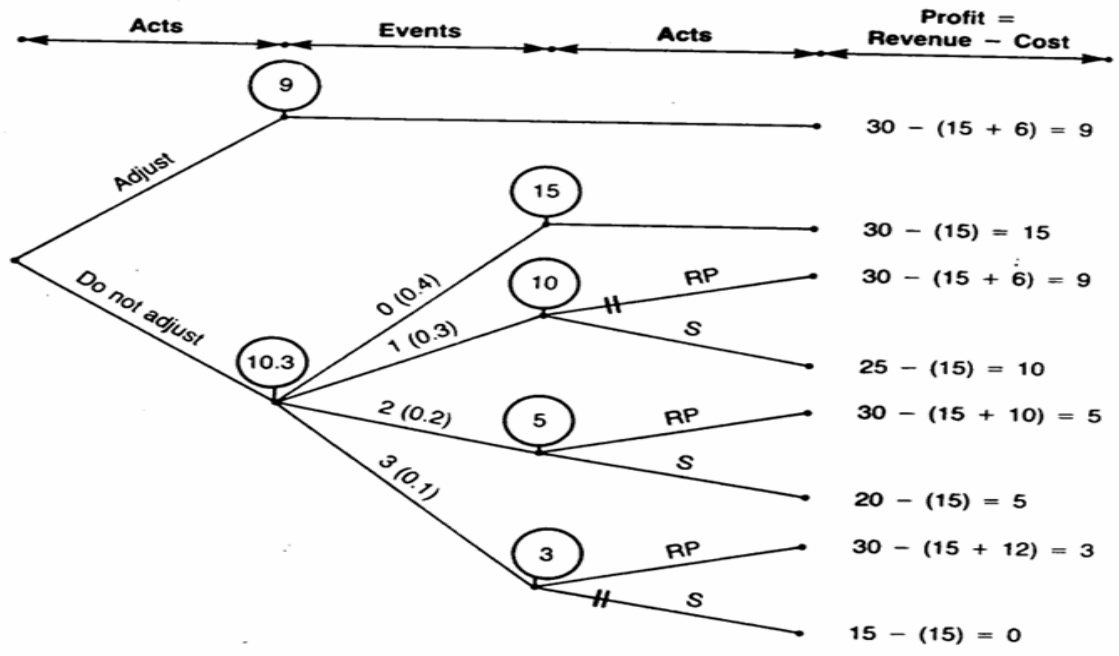
Number of defectives in batches of 3 items	Probability
0	0.4
1	0.3
2	0.2
3	<u>0.1</u>
	1.0

Defective items may be sold as scrap for \$5 each, or may be reprocessed at the following cost schedule:

Number of reprocessed items	Total reprocessing cost
1	\$ 6
2	10
3	12

Reprocessed items can be sold as good items. Should the machine be adjusted before each batch is produced? The figure below shows the decision tree for this problem.

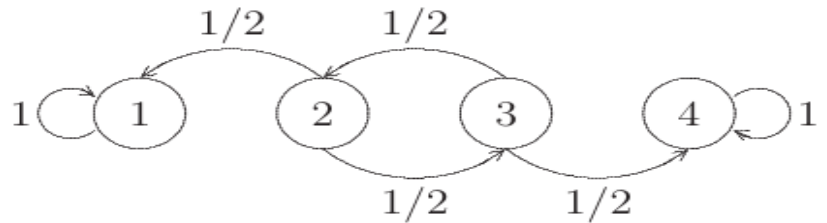
(7 Marks)



Q3.

a)

Suppose $\{X_t : t \geq 0\}$ has the following transition diagram:



Find the vector of hitting probabilities for state 4.

(10 Marks)

b) Glee-flea hops around on a triangle. At each step he moves to one of the other two vertices at random. What is the expected time taken for Glee-flea to get from vertex 1 to vertex 2?

(10 Marks)

Q4.

a) Suppose a university outreach program provides two services: campus tour days and publicity mailing days. The outcomes and requirements of both services are shown here:

Service	Cost	Hours	# applicants	# donors
Tour	100	32	10	4
mailing	500	16	2	6
requirement	≤ 2600	≤ 256	Maximize	Maximize

Graph the feasible region, find the two optimal points, and highlight the segment between them.

(10 Marks)

- b) Convert your design space from question (a) above into criterion space. Highlight the segment between the two optimal points from question (a). **(10 Marks)**

END

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