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MAIN EXAMINATION

JANUARY - APRIL 2015 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES (PHYSICS)

PHY 304: STATISTICAL PHYSICS

Date: April 2015	**Duration: 2 Hours**
Instructions: Answer Question ONE and any other TWO Questions.	

Important Standard Integral

$$I_{(o)} = \alpha^{-1/2}\int_0^\infty e^{-y^2}dy = \frac{\sqrt{\pi}}{2}\alpha^{-1/2}$$

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^{\infty} e^{-x^2}dx = 2\int_0^{\infty} e^{-x^2}dx$$

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

Q1. a) Consider a situation in which the total number of steps is N = 3.

i) Find the probability that all the three steps are to the right. Illustrate using n_1 for number of steps to the right,
n_2 for number of steps to the left, and
m for net displacement ($m = n_1 - n_2$)

(3 marks)

ii) Find the probability that two steps are to the right while the third step is to the left.

(2 marks)

b) State the essential ingredients necessary for analyzing systems of very many particles.

(4 marks)

c) Consider a system in equilibrium, which is isolated so that its total energy is known to have constant value in some range between E and E + δ E. Let Ω(E) denote the total number of states of the system in this range. Suppose that there are among these states a certain number $\Omega(E; y_k)$ of states for which some parameter y of the system assumes the value y_k.

i) What is the probability $P(y_k)$ that the parameter y of the system assumes the value y_k.

(1 mark)

ii) Find the mean value of the parameter y for this system in the ensemble.

(1 mark)

d) i) Give an elaborate description of constraints.

(3 marks)

ii) What happens if some constraints of an isolated system are removed?

(2 marks)

e) Consider two macroscopic systems A and A^1 in thermal interations with with each other, with respective energies E and E. Let the energy scales be subdivided into equal small intervals of respective magnitudes δE and $\delta E^{/}$.

i) Assuming that the systems are not thermally insulated from each other write down the expression for the combined system $A^{(0)}$ and the corresponding total energy $E^{(0)}$.

(2 marks)

ii) Let Ω(E) and $\Omega'(E')$ denote the number of states accessible to A and A' in thermal interation with each other, with respective. And $\Omega^0(E)$ denote the number of states accessible to $A^{(0)}$. What is the probability P(E) of finding the combined system in a configuration where A has an energy between E and E+δE.

(2 marks)

f) i) What is meant by canonical distribution?

(1 mark)

ii) Write the expression for canonical distribution in explicit form.

(1 mark)

iii) The sums over all accessible states r of a system, irrespective of their energy is called the partition function. Write the expression for the partition function.

(1 mark)

g) i) State the equipartition theorem.

(1 mark)

ii) List three applications of the equipartition theorem.

(3 marks)

iv) Given that the energy of a one-dimensional harmonic oscillator which is in equilibrium with a heat reservoir at absolute temperature T is given by

$$E = \frac{p^2}{2m} + \frac{1}{2}K_0 x^2$$

By equipartition theorem, find the mean total energy.

(2 marks)

Q2. a) Consider a system with energies $E_0 = -\mu\beta$ and $E_1 = \mu\beta$.

i) Find the partition function of the system.

(2 marks)

ii) Find the average energy of the system as a function of temperature.

(5 marks)

b) A box is separated by a partition which divides its volume in the ratio 3:1. The larger portion of the box contains 1000 molecules of Ne gas; the smaller, 100 molecules of He gas. A small hole is punctured in the partition, and one wait until equilibrium is attained.

i) Find the mean number of molecules of each type on either side of the partition.

(2 marks)

ii) What is the probability of finding 1000 molecules of N_e gas in the larger potion and 100 molecules of H_e gas in the smaller (i.e, the same distribution as in the initial system)?

(10 marks)

Q3. a) A simple harmonic one-dimensional oscillator has energy levels given by $E_n = (n + ½)\hbar\omega$, where ω is the characteristic frequency of the oscillator and where the quantum numbers n can assume the possible integral values n = 0, 1, 2….. Suppose that such an Oscillator is in thermal contact with a heat reservoir at temperature T low enough so that $\frac{KT}{\hbar\omega} << 1$.

i) Find the ratio of the probability of the oscillator being in the first state the probability of its state being in the ground state.

(6 marks)

ii) Assuming that only the ground state and the first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature.

(4 marks)

b) Given the probability distribution

$$P_{(x)}dx = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2} dx$$

Where $\mu = (p-q)Nl$ and $\sigma = 2\sqrt{Npql}$

i) Show that the probability of a particle being anywhere is unity.

(2 marks)

ii) Calculate the mean value, $\bar{x}$ **(3 marks)**

iii) Find the dispersion $\overline{\Delta x^2}$ **(4 marks)**

Q4. a) When a system A is in thermal contact with a heart reservoir, or when only its mean energy is known, the systems in the respective canonical ensemble are distributed over their accessible state in accordance with the canonical distribution.

$$P_r = \frac{e^{-BE_r}}{\sum_r e^{-BE_r}}$$

i) Derive the expression for the mean energy $\bar{E}$ in terms of the partitions functions Z.

(6 marks)

ii) Determine the dispersion of energy

$$\left(\overline{\Delta E}\right)^2 = \bar{E}^2 - \bar{E}^2$$

(7 marks)

iii) Show that the mean pressure for the system is given by

$$\bar{p} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial v}$$

Where V is volume of the system.

(7 marks)

Q5. a) Consider the random walk problem in which p = q and m = $n_1 - n_2$ is the net displacement to the right. After a total of N steps, calculate the following mean values;

i) $\bar{m}$ **(7 marks)**

ii) $\bar{m}^2$ **(7 marks)**

b) A coin is tossed 400 times. By using the Gaussian approximation find the probability of getting 215 heads.

(6 marks)

END