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MAIN EXAMINATION

SEPTEMBER - DECEMBER 2023

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

MAT 460: STOCHASTIC PROCESS

DATE: DECEMBER 2023	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1.

a. Define the following:
   i. Stochastic Process **(2 Marks)**
   ii. Ergodic Markov Chain **(2 Marks)**
   iii. Counting process **(2 Marks)**
   iv. Birth death process **(2 Marks)**

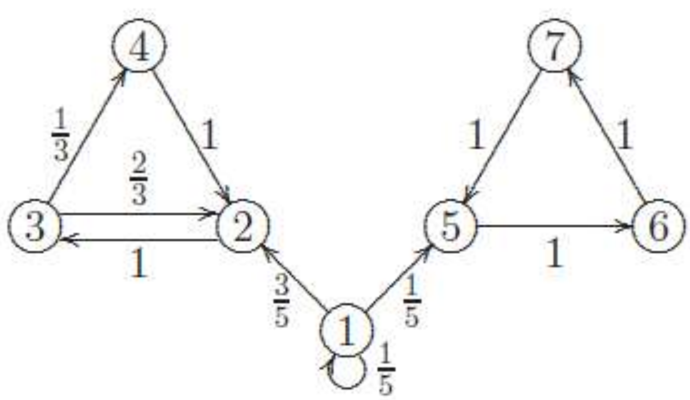
b. Differentiate between a transient and recurrent state **(2 Marks)**

c. When is a random process said to be stationary? **(2 Marks)**

d. In an exam, 10 multiple choice questions are asked where only one out of four questions are correct. Find the probability of getting 5 out of 10 questions correct in an answer sheet. **(5 Marks)**

e. Let $a_k = k$ for $k = 0,1,2,3, \ldots,$. Show that the generating function for the sequence $\{a_k\}$ is given by $A(s) = \frac{s}{(1-s)^2}$ **(5 Marks)**

f. Consider the Markov chain below



   i. Is this chain irreducible? Why? **(2 Marks)**

ii. Identify the communicating classes. **(3 Marks)**

iii. Let $X_0 \sim \left(\frac{3}{4}, 0, \frac{1}{4}, 0,0,0,0\right)$. What is the probability of the trajectory 1,2,3,2,3,4 **(3 Marks)**

Q2.

a. The number of customers arriving at a grocery store can be modelled by a Poisson process with intensity $\lambda = 10$ customers per hour. Find the probability that there are 3 customers between 11.00 and 11.20 and 7 customers between 11.30 and 12.10 **(9 Marks)**

b. Suppose the process $\{X_t : t \geq 0\}$ be a Poisson process having rate $\lambda = 8$. Find $P\{X_{1.5} = 10, X_{3.5} = 18, X_5 = 30\}$ **(11 Marks)**

Q3.

a. Given the PMF of a Poisson random variable as

$$p(x) = \frac{e^{\lambda}\lambda^x}{x!}$$

i. Determine the generating function **(5 Marks)**

ii. Use the generating function obtained in part (i) above to determine the mean and variance of the Bernoulli random variable. **(9 Marks)**

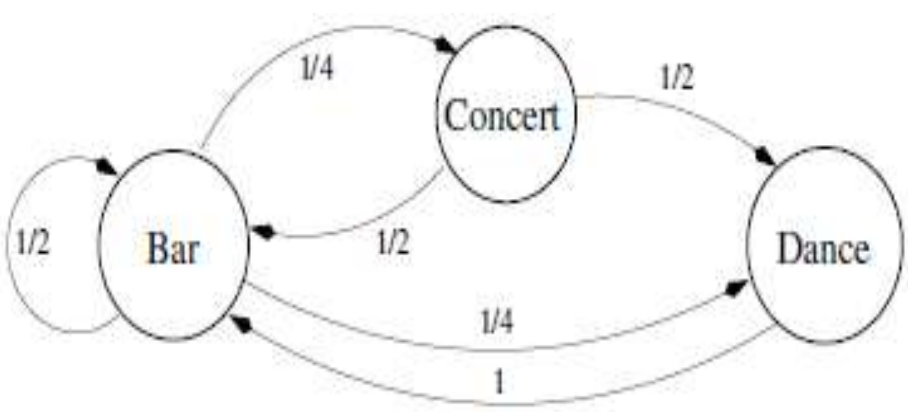
b. When Stéphane plays chess against his favorite computer program, he wins with probability 0.60, loses with probability 0.40. Assume independence.

i. Find the probability that Stéphane's first win happens when he plays his third game. **(3 Marks)**

ii. Find the probability that Stéphane wins 7 games, if he plays 10 games. **(3 Marks)**

Q4.

Consider the following Markov chain where the bar is state 1, the concert is state 2 and dance is state 3.



a. Given that the process starts in state 1 and $X_0 \sim \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$, find the probability distribution of X_2. **(5 Marks)**

b. Find a stationary distribution for the Markov chain **(15 Marks)**

Q5.

Given the probability of success $P(x = k) = P_k, k = 0,1,2, \ldots.$ has a generating function $P(s)$. Show that $Q(s) = \frac{1-P(s)}{1-s}$ where $Q(s)$ is the generating function of the probability of failure $q_k = P(X > k)$. Hence show that $E(x) = Q(1)$ and $Var(x) = 2Q'(1) + Q(1) + [Q(1)]^2$.

(20 Marks)

END