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MAIN EXAMINATION

SEPTEMBER - DECEMBER 2023

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

MAT 335: METHODS OF APPLIED MATHEMATICS I

DATE: DECEMBER 2023	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1.

a) Classify the following 2nd order partial differential equations as parabolic, elliptic or hyperbolic

i) $yU_{xx} + U_{yy} = 0$ **(2 Marks)**

ii) $U_{xx} + 2\sin x\, U_{xy} - cos^2 x U_{yy} - \cos x\, U_y$ **(2 Marks)**

iii) $y^2U_{xx} - 2xyU_{xy} + x^2U_{yy} = \frac{y^2}{x}U_x + \frac{x^2}{y}U_y$ **(2 Marks)**

b) Given a Beta function $\beta(m,n) = \int x^{m-1}(1-x)^{n-1}dx$, prove that $\beta(m,n) = \beta(n,m)$

(3 Marks)

c) Prove that the following mathematical statements are true:

i) $L\{a\} = \frac{a}{s}$ **(2 Marks)**

ii) $L\{\sinh at\} = \frac{a}{s^2-a^2}$ **(4 Marks)**

d) Find the general solution of the first order differential equation by use of transforms

$\frac{dx}{dt} + 2x = 10e^{3t}$ given that $x(0) = 6$ **(6 Marks)**

e) Find the value of $\Gamma\left(\frac{7}{2}\right)$ **(5 Marks)**

f) Use Gamma function to evaluate $\int_0^\infty x^3 e^{-4x} dx$ **(4 Marks)**

Q2.

a) Apply First Shift Theorem to evaluate

$L\{tSin\ 2t\}$ **(4 Marks)**

b) Solve the boundary value problem

$$\frac{d^2x}{dt^2} - 3\frac{dy}{dx} + 2x = 2e^{3t}, \quad x_0 = x(0) = 5, \quad x_1 = \frac{d}{dx}(x(0)) = 7$$ **(10 Marks)**

c) Evaluate $\int_0^1 x^4\sqrt{1-x^2}\,dx$ **(6 Marks)**

Q3.

a) Consider heat conduction in a thin metal bar of length l with insulated sides. Let the ends $x = 0$ and $x = l$ be held at temperature $U = 0^0C$ for all $t > 0$. In addition, suppose that the temperature distribution at $t = 0$ is $U(x, 0) = f(x)$, $0 \le x \le l$. Determine the temperature distribution in the bar at some subsequent time $t > 0$. **(12 Marks)**

b) Find $L^{-1}\left\{\frac{5s^2-23s+26}{s^3-6s^2+11s-6}\right\}$ **(8 Marks)**

Q4.

a) Given the Fourier Series as $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, determine the values of a_0, a_n and b_n. **(10 Marks)**

b) Find the Fourier series for the function

$$f(x) = \begin{cases} -x; & -\pi < x < 0 \\ 0; & 0 < x < \pi \\ f(x) & = f(x+2\pi) \end{cases}$$ **(10 Marks)**

END