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GABA CAMPUS - ELDORET

MAIN EXAMINATION

JANUARY– APRIL 2023 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 335: METHODS I

DATE: April 2023	**Duration: 2 Hours**
Instructions: Answer Question ONE and any other TWO Questions	

Q1.

a) Prove that $L\{t\} = \frac{1}{s^2}$ **(3 Marks)**

b) Apply First Shift Theorem to evaluate:

i) $L\{2e^{3t}Cos\ 3t\}$ **(3 Marks)**

ii) $L\{t^2 Sinh\ 3t\}$ **(5 Marks)**

c) Apply L'Hospital Rule to determine

$$L\left\{\frac{\sin at}{t}\right\}$$ **(5 Marks)**

d) Evaluate $L^{-1}\left\{\frac{5s^2-23s+26}{s^3-6s^2+11s-6}\right\}$ **(5 Marks)**

e) Show that $\Gamma(x+1) = x\Gamma(x)$ **(4 Marks)**

f) Prove that $\beta(m,n) = \beta(n,m)$ **(2 Marks)**

g) Evaluate $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ **(3 Marks)**

Q2.

a) Classify the following equations as hyperbolic, parabolic or elliptic

i) Heat equation $\alpha U_{xx} - U_t = 0$ **(2 Marks)**

ii) Laplace equation $U_{xx} + U_{yy} = 0$ **(2 Marks)**

iii) Wave equation $c^2U_{xx} - U_{tt} = 0$ **(2 Marks)**

b) The displacement of a vibrating string is described by the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With the boundary conditions

$x = 0$, $\quad u(0,t) = 0$

$x = l$, $\quad u(l,t) = 0$

And the initial conditions: $t = 0$, $\quad u(x,0) = \phi(x)$

Apply the method of separation of variables to find the general solution.

(10 Marks)

c) Find the value of $\Gamma\left(\frac{5}{2}\right)$ **(4 Marks)**

Q3.

a) Use Gamma function to evaluate $\int_0^\infty x^3 e^{-4x} dx$ **(4 Marks)**

b) Find the general solution of the first order differential equation by use of transforms

$\frac{dx}{dt} + 2x = 10e^{3t}$ given that $x(0) = 6$ **(6 Marks)**

c) Solve the boundary value problem

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}$$

$x(0) = 5$ and $x'(0) = 7$ **(10 Marks)**

Q4.

a) Given a Beta function $\beta(m,n) = \int x^{m-1}(1-x)^{n-1} dx$, prove that $\beta(m,n) = \beta(n,m)$

(3 Marks)

b) Use Beta function to evaluate $\int_0^1 x^4 \sqrt{1-x^2}\, dx$ **(5 Marks)**

c) Find the Fourier series for the function

$$f(x) = \begin{cases} -x; & -\pi < x < 0 \\ 0; & 0 < x < \pi \\ f(x) & = f(x+2\pi) \end{cases}$$

(12 Marks)

END